Undergrad Physics Review

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1 Classical Mechanics

1.1 Kinematics

	Linear	Angular
Displacement	Δx	$\Delta\Theta$
Velocity	$v = \frac{dx}{dt} = \dot{x}$	$\omega = \frac{d\Theta}{dt} = \dot{\Theta}$
Acceleration	$a = \frac{dv}{dt} = \dot{v} = \ddot{x}$	$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\Theta}$
Constant-acc.	$\Delta x = v_{av}\Delta t$	$\Delta\Theta = \omega_{av}\Delta t$
Equations	$v_{av} = \frac{1}{2}(v_0 + v)$	$\omega_{av} = \frac{1}{2}(\omega_0 + \omega)$
	$v = v_0 + 2at$	$\omega = \omega_0 + \alpha t$
	$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\Theta$
	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$	$\Theta = \Theta_0 + \omega_0 t + \frac{1}{2}\alpha t$
Mass	m	I
Momentum	$p = mv = m\dot{x}$	$L = \vec{r} \times \vec{p} = I\omega = I\dot{\Theta} = mvrsin\theta$
Force	F	τ
Kinetic energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\dot{\Theta}^2 = \frac{1}{2}\frac{L^2}{I}$
Power	$P = Fv = F\dot{x}$	$P = \tau\omega = \tau\dot{\Theta}$
Newton's 2nd Law	$F_{net} = \dot{p} = ma = m\ddot{x}$	$\tau_{net} = \frac{dL}{dt} = I\alpha = I\ddot{\Theta} = Fl = \vec{r} \times \vec{F} = \vec{N}$

- **center of mass**: $R_{CM} = \frac{1}{M}\int rdm$
- **circular motion**: $F = ma_{\perp} = m\frac{v^2}{r} = m\frac{(r\omega)^2}{r} = mr\omega^2$
- **collisions**: elastic: KE conserved; inelastic: KE not conserved

- **pressure**: $P = F/A = \frac{-1}{k}\frac{dy}{dx}$
- **Impulse** $= F\Delta t = \int_{t_1}^{t_2} F dt = \Delta \vec{p}$
- **friction**: $f = \mu_k N$ for kinetic friction and $f \leq \mu_s N$ for static friction
- **springs**: $F_s = -k|l - l_0| = -kx$ with e.o.m: $F = m\ddot{x} = -kx \Rightarrow x(t) = Asin(\omega t + \phi)$ where $\omega = \sqrt{\frac{k}{m}}$
- **Virial theorem**: (n-power law force) $< T >= \frac{n+1}{2} < U >$; for gravity, n=-2

1.2 Work/Energy/Power

- $W = F \cdot x = \int_{x_1}^{x_2} F_x dx = -\Delta U$ (F conservative)
- $Power = P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = \frac{E}{t} = (W = J/s)$
- $W_{NET} = \Delta KE$
- Flux $= S = Power/Area$

1.3 Gravity/Orbital Motion

- $F_g = mg = G\frac{m_1 m_2}{r^2}$
- $U_g = mgh = -G\frac{m_1 m_2}{r}$
- Kipler's 3rd: $T^2 = \frac{4\pi^2}{GM} r^3$
- $E_{tot} = \frac{-GMm}{2r} = -\frac{GMm}{2a}(ellipse)$ where a is one half the major axis length.
- $\mathcal{L} = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$
- $l = \mu r^2 \dot{\theta}$
- $E = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) + U(r) = \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$
- $\frac{d^2}{d\Theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{-\mu r^2}{l^2} F(r)$where $l=$ angular momentum

1.4 Rotation

- torque tip: all forces about the point in question are irrelavent
- moment of inertia $\equiv I = \int r^2 dm = \sum_i m_i r_i^2$
- rolling (w/o slipping) condition: $v = r\omega$
- $a = \alpha R_{CM}$
- force from translational acc: $m\ddot{R}$
- from from ang. acc: $m\dot{\omega} \times \vec{r}$
- centrifugal force: $m\vec{\omega} \times (\vec{\omega} \times \vec{r}) = (m\omega^2 r$ for ω normal to radius vector)
- Coriolis force: $2m\vec{\omega} \times \vec{v}$

Moments of Inertia, I

cyl shell/axis$=MR^2$	rod/center$=\frac{1}{12}ML^2$
solid cyl./axis$=\frac{1}{2}MR^2$	rod/end$=\frac{1}{3}ML^2$
hollow cyl./axis$=\frac{1}{2}M(R_1^2 + R_2^2)$	sph. shell$=\frac{2}{3}MR^2$
cyl. shell/center$=\frac{1}{2}MR^2 + \frac{1}{12}ML^2$	solid sph.$=\frac{2}{5}MR^2$
solid cyl./center$=\frac{1}{4}MR^2 + \frac{1}{12}ML^2$	rect/$\perp$ face$=\frac{1}{12}M(a^2 + b^2)$

1.5 Pendulums

- normal pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$
- physical pendulum: $T = 2\pi\sqrt{\frac{I}{MgD}}$
- torsional pendulum: $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$ where $\omega = \sqrt{\frac{\kappa}{I}}$
- Parallel axis theorem for I: $I = I_{CM} + Mh^2$ where h is the distance from CM to axis

1.6 Waves

- $v = \lambda f$
- $\omega = 2\pi f = kv$

- $k = \frac{2\pi}{\lambda}$
- $v_p = \frac{\omega}{k} \equiv$ phase velocity of a wave
- $v_g = \frac{d\omega}{dk} \equiv$group velocity of wave
- harmonic wave: $y(x,t) = Asin(kx - \omega t)$ where A is the amplitude, k is the wave number, and ω is the angular frequency.

on a string: $v_p = \sqrt{\frac{T}{\mu}}$where T is the tension(force) and $\mu = m/L$

$$\text{fundamental modes: } f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{nv_p}{2L}$$

$$L = \frac{n\lambda_n}{2}$$

sound: $v = \sqrt{\frac{\gamma P}{\rho}}(gas) = \sqrt{\frac{B}{\rho}}$where $B = \frac{P}{\Delta V/V}$

$$\Delta P = P_{max} sin(kx - \omega t - \delta)$$

$$P_{max} = \rho v \omega S_{max} \text{where } S_{max} \text{ is the max displacement}$$

$$\text{Intensity} = I = \frac{\Delta P_{max}^2}{2\rho v}$$

in tube $f_n = n\frac{v}{2L}$ where $n = 1, 2, 3, ...$ (both ends open)

$$f_n = n\frac{v}{4L} \text{ where } n = 1, 3, 5, ... \text{ (one end open)}$$

1.7 Fluids

- $I_v = A_1 v_1 = A_2 v_2 =$ constant = current flow (continuity equation)
- Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 =$ constant; where y is the height of the pipe(fluid CM), v is the speed of the fluid, and P is the pressure on the fluid.
- $P = P_0 + \rho g h$ (relates the pressure at the top(0) and bottom of a column of fluid).
- Buoyant force: $F_B = B = w$ =weight of the fluid displaced; $\rho_{water} = 200\text{kg/m}^3$
- Venturi effect: When the speed of a fluid decreases, the pressure drops.

1.8 Eqns of Motion

- Lagrange Equation: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$
- Griffith's method: $A_{jk} = \frac{\partial^2 U}{\partial q_j \partial q_k}|_0$ and $T = \frac{1}{2}\sum_{j,k} m_{jk}\dot{x}_j\dot{x}_k$ then solve the scular equation $det(A - \omega^2 m) = 0$

- Hamilton: $\dot{q} = \frac{\partial H}{\partial p}; -\dot{p} = \frac{\partial H}{\partial q}$

1.9 Oscillatory Motion

- $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- $F = -\frac{dU}{dx}$
- $-k = F' = -U''$ and $\omega = \sqrt{\frac{k}{m}}$ and $T = \frac{2\pi}{\omega}$(small oscillations)
- Damped osciallations: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$ where $\beta = b/2m$ is the damping parameter
- w/ solutions: $x(t) = e^{-\beta t}[A_1 e^{(\beta^2-\omega_0^2)^{1/2}t} + A_2 e^{-(\beta^2-\omega_0^2)^{1/2}t}$ where $\omega_0 = \sqrt{k/m}$

1. $\omega_0^2 > \beta^2$: underdamping
2. $\omega_0^2 = \beta^2$: critical damping
3. $\omega_0^2 < \beta^2$: overdamping

- Damped driven oscillations: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = Acos(\omega t)$

1.10 Material Properties

- Elasticity: $Y(Young's modulus) = \frac{F/A}{\Delta L/L_0} = \frac{stress}{strain}; B = \frac{-F/A}{\Delta V/V_0}$
- $P = P_0 + \rho gh$ (relates the pressure at the top(0) and bottom of a column of fluid).
- Length expansion: $\Delta L = \alpha L_0 \Delta T$

1.11 Scattering

- Closest approach/minimum p-α distance occurse when PE=KE
- Rutherford scattering: $\frac{d\sigma}{d\Omega} = \frac{|k/4E|^2}{sin^4(\theta/2)}$
- Estimate of reflection of charged sphere of radius R: $tan\theta = \frac{kq_\alpha Q}{R(\frac{1}{2}m_\alpha v^2)}$
- Impact parameter and scattering angle: $b = \frac{kq_\alpha Q}{m_\alpha v^2}cot\frac{1}{2}\theta$

1.12 Dopplar Effect

- $f' = f_0 \frac{(1 \pm u_r/v)}{(1 \mp u_s/v)}$ where u_r and u_s are the speed of the receiver and source relative to the medium, respectively. The correct choices for the plus or minus sign are most easily obtained by remembering that the frequency increases when the source and receiver are moving toward each other and decreases when they are moving away from each other.

- $f' = \frac{v'}{\lambda}$ where v' is the speed of the waves relative to the observer and λ is the wavelength of the medium. $v' = v \pm u_w$ if the medium is moving at speed u_w

2 Relativity

- $E = \sqrt{c^2p^2 + m^2c^4}$

- $p = \gamma mv$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

- $E = \gamma mc^2$

- $pc = \beta E$ where $\beta = \frac{v^2}{c^2}$

- $\vec{p}c^2 = E\vec{v}$

- $T = \gamma mc^2 - m_0c^2$

- $E \approx pc$ for $E_0 >> m_0c^2$

- 4-vector: $(E/c, \vec{p})$

- squares of 4-vectors are invariant:

$$p_\mu p^\mu = \left(\frac{E}{c}\right)^2 - (p_x^2 + p_y^2 + p_z^2) = \left(\frac{E'}{c}\right)^2 - (p_x'^2 + p_y'^2 + p_z'^2) = p'_\mu p'^\mu$$

- time dilation: $\Delta t' = \gamma \Delta t$ where t is the proper time (frame are rest w.r.t clock) - moving clocks run slow

- length contraction: $\gamma \Delta L' = \Delta L$

- relativistic dopplar effect: $f' = \sqrt{\frac{1 \pm V/c}{1 \mp V/c}}$ where the sign: top=approaching and bottom=receding
- velocities: $u = \frac{u' + V}{1 + Vu'/c^2}$

3 E&M

Maxwell's Equations

Free Space	In Media	with definitions...
$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	$\nabla \cdot \mathbf{D} = \rho$	$D = \epsilon E$, $D = \epsilon_0 E + P$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$H = B/\mu$, $H = \frac{1}{\mu_0} B - M$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$	$P = \epsilon_0 \chi_e E$ (polarization)
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$M = \chi_m H$
		$\nabla \cdot D = 0 \Rightarrow D_z$ is continuous
		$\nabla \times E = 0 \Rightarrow E_{\|\|}$ is continuous

Integral Forms

$\int E \cdot dl = \mathcal{E} = -\frac{d\Phi}{dt}$ (Faraday's Law) where $\Phi = \int B \cdot da = LI$ (I is current)

$\int E \cdot da = \frac{Q_{in}}{\epsilon_0}$ (Gauss's Law)

$\int B \cdot dl = \mu_o I_{enc} + \frac{1}{c^2} \frac{d\Phi_E}{dt}$ (Ampere's Law)

General Equations

$F_e = \frac{k q_z q_2}{r^2}$	Capacitance	B-Fields
$E = \frac{F}{q} = -\nabla V$	$Q = CV$	$B = \nabla \times A$
$V = -\int_{(-)}^{(+)} E \cdot dl$	$W = \frac{1}{2} C V^2$	$B = \frac{\mu_0}{4\pi} I \oint \frac{dl \times r}{r}$, $d\vec{B} = kI \frac{d\vec{l} \times \vec{x}}{\|\vec{x}\|^3}$
$F = q(E + v \times B)$ (Lorentz)	$C_{pp} = \frac{\epsilon_0 A}{d}$	$F_{rod} = I\vec{L} \times \vec{B}$
$V = IR$, $P = IV$	$C_{sphere} = 4\pi\epsilon_0 A$	$B_{toroid} = \frac{\mu_0 N I}{2\pi r}$, $B_{circle} = \frac{\mu_0 I}{2r}$
$I = \frac{dQ}{dt}$	$C_{shell} = 4\pi\epsilon_0 \frac{ab}{a-b}$	$B_{solenoid} = \mu_0 (N/L) I$
$J = \sigma E = nqv_d$ (current density)	$C_{coax} = \frac{2\pi\epsilon_0 L}{ln(a/b)}$	$\mathcal{E}_{motional} = -vBl$
$v_d = \frac{\tau}{m} qE$ where τ=collision time	w/ dielec.: $V = V_0/\kappa$ and $C = \kappa C_0$	mag. moment: $\vec{\mu} = I\vec{A}$, $\tau = \mu \times B$
$v_d = \mu \vec{F}$ where μ is the mobility	$\tau = v_{rms}/l$	
$\rho = \frac{1}{\sigma}$; E-fields in matter:	$\sigma_b = \vec{P} \cdot \hat{n}$, $\rho_b = -\nabla \cdot \vec{P}$	$W = \frac{1}{2} \int \vec{E} \cdot \vec{D} d^3 x$

Dipoles

$$p = qd$$

$$\tau = p \times E$$

$$W = -p \cdot E$$

$$\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$$

Inductance

$$\Phi_2 = M_{21} I_1$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|r - r'|}$$

$$M_{21} = M_{12}$$

$$\Phi = LI$$

$$W = LI^2$$

EM Equations

Energy: $U = \frac{1}{2} \int (\epsilon_0 E^2 + \frac{1}{\mu} B^2) d\tau$

Field Momentum: $P = \epsilon_0 \int (E \times B) d\tau$

Poynting vector (intensity) $\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$

Larmor formula: $P = \frac{\mu_0}{6\pi c} q^2 a^2$

=Power radiated by charge with acc. a

$W_{mag} = \frac{1}{2\mu} \int_{all} B^2 d\tau$, $W_{elec} = \frac{\epsilon_0}{2} \int E^2 d\tau$

$|E| = c|B|$

$F = \dot{P}$

$n = \sqrt{\frac{\mu\epsilon}{\mu_0 \epsilon_0}}$

Wvgd: $k^2 = \mu_0 \epsilon_0 \omega^2 - \gamma^2$

wh/ $\gamma^2 = \pi^2 \left(\frac{n^2}{a^2} + \frac{m^2}{b^2} \right)$

$TE_{mode} : \omega_{cutoff} = c\gamma$

$I = \frac{EB}{2\mu_0}$ (intensity)

enrgy density: $u_E = \frac{1}{2} \epsilon_0 E^2$, $u_B = \frac{1}{2\mu_0} B$

mom. and rad. pres.: $p = \frac{U}{c}$, $P = \frac{S}{c}$

Helmholtz W. Eq.: $\nabla^2 (E|B) = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} (E|$

K = current/area perp. to flow

dispersion relation: $\omega^2 = \omega_p^2 + c^2 k^2$

$$V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-(l+1)}) P_l(cos\theta)$$

$$\sigma(\Theta) = -\epsilon_0 \frac{\partial V}{\partial n} (normal D at surface)$$

Circuit Equations

LR Circuit: $I(t) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$ where $\tau = L/R$

LC Circuit: $I = -\omega Q_{max} sin(\omega t + \phi)$ where $\omega = 1/\sqrt{LC}$

$$E = \frac{Q_{max}^2}{2C} cos^2 \omega t + \frac{L I_{max}^2}{2} sin^2 \omega t$$

$$P_{av} = I_{rms} V_{rms} cos\phi = I_{rms}^2 R$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ with $X_L = X_C$

Tansformers: $\frac{V_2}{N_2} = \frac{V_1}{N_1}$

If ϕ is + then I lags V

AC circuits

$X_L = \omega L$, $X_C = \frac{1}{\omega C}$

$$tan\phi = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{rms} = \frac{V_{rms}}{Z}$$

If ϕ is - then I leads V

Derivation of Wave Equations

$$\nabla \cdot E = 0 \ ; \ \nabla \cdot B = 0 \ ; \ \nabla \times E = -\frac{\partial B}{\partial t} ; \ \nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E = -\nabla \times \frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times B) = -\frac{\partial}{\partial t}(\frac{1}{c^2}\frac{\partial E}{\partial t})$$

$$-\nabla^2 E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \Rightarrow \nabla^2 E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

and the same for B.

Now assume solution of type $e^{i(kx-\omega t)}$ and use the tricks: $\nabla \rightarrow ik$ and $\frac{\partial}{\partial t} \rightarrow -i\omega$

$$-k^2 E = \frac{-\omega^2}{c^2} E$$

$$k^2 = \frac{\omega^2}{c^2} = \mu_0 \epsilon_0 \omega^2$$

4 Thermodynamics

Basic Thermo

$dU = TdS - PdV + \mu dN$	$W = \int pdV = nRTln\left(\frac{V_f}{V_i}\right)_{T,P}$	$c_{water} = 4.18$ kJ/kg$\cdot K$
$dG = -SdT + VdP$	equilibrium$\equiv$max S	$1L = 10^{-3} m^3$
$dS = \frac{dQ}{T} = \frac{CdT}{T}$	max W <-> $\Delta S = 0$	$1L \cdot atm = 101.3J$
1st Law: $dU = \delta Q - \delta W$	$dW = pdV = Fdx$	$1atm = 101.3 \times 10^3 N/m^2$
$Q = mc\Delta T = nC_V \Delta T = nC_P \Delta T$	W=area under PV curve	1 cal = 4.184 J
$Q = mL_{v,f}$ (latent heat, $T = 0$)	$\frac{dQ}{T} = C_v \frac{dT}{T} + nR\frac{dV}{V}$	$R = 0.0826$ L$\cdot$atm/mol$\cdot$K= kN_A
$c = C/m$		

Gases

$pV = nRT = NkT$	$\gamma = \frac{C_P}{C_V}$	adiabatic: PV^γ =constant, $\Delta Q = 0$
$U = \frac{3}{2}NkT = KE$	$C_P > C_V$	$W_{adia} = -C_V\Delta T$
$C = \frac{dU}{dT}$	$C_P = C_V + nR$	$\Delta Q_{adia} = 0$
$dQ = nC_V dT$, $dQ = nC_P dT$, $dQ = mcdT$	for a cmplx mol., $\gamma \to 1$?	isobaric: P is constant, $W = pdV$
	isothermal: $\Delta U = 0$	$W_{isothermal} = \int pdV = nRTln(V_2/V_1)$
		constant V: $W = 0$

- **Equipartition theorem**: When a substance is in equilibrium, there is an average energy of $\frac{1}{2}kT$ per molecule of $\frac{1}{2}RT$ per mole associated with each degree of freedom.
- monatomic gas: $\gamma = 5/3$; $U = \frac{3}{2}nRT$; $C_V = \frac{3}{2}nR$; $C_P = \frac{5}{2}nR$
- diatomic gas: $\gamma = 7/5$; $U = \frac{5}{2}nRT$; $C_V = \frac{5}{2}nR$; $C_P = \frac{7}{2}nR$
- Dulong and Petite Law: for a 3-D crystal w/ springs - 6 degrees of freedom: $C_M = 3R$
- The Eq.T only demonstrates a linear dependance on T; one of the major failings of classical theory. To correct this, Planck replaced kT with $\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}$.
- Quantum corrections to Eq.Theory: For a diatomic molecule: T<30K: only translational degrees of freedom active: $C_V = \frac{3}{2}R$; for $30K < T < 3000K$: rotational modes are additionally excited: $C_V = \frac{5}{2}R$; for $3000K < T < 12,000K$: vibrational modes are excited as well: $C_V = \frac{7}{2}R$; and for $T > 12,000$: C_V increases approximately linearly due to atomic disruptions.

Heat and Power and Radiation

Net power radiated by and object: $I_{net} = e\sigma A(T^4 - T_0^4)$	$\sigma = 5.67 \times 10^{-8} W/m^2K^4$
power/unit area (total emmisivity) $=\sigma T^4 = \int^\infty E_f df$	$R = \frac{1}{4}cU$
Wien's displacement Law: $\lambda_{max}T = 2.89 \times 10^{-3} m \cdot K$	Rayleigh-Jeans Law (classical): $u(\lambda) = \frac{8\pi kT}{\lambda^4}$
where λ_{max} is the wavelength corresponding to the max I	where $u(\lambda)$=spectral distro. func. $\Rightarrow$UV cat.
$T_{sun} = \frac{2.89\times10^{-3} m\cdot K}{500\times10^{-9} m} = 5800K$	Power/A from sun at earth=1400W/m^2

Planck's rad. Law: (R-J Law)$\times$(#osc. in interval $d\lambda$)=$(8\pi\lambda^{-4}) \times \left(\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}\right) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1} = u(\lambda)$

or $u_\omega = \frac{\hbar}{\pi^2c^2}\frac{\omega^3}{e^{\hbar\omega/kT}-1}$

More Heat

Thermal current: $I = \frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$, $\Delta T = IR$, $R = \frac{\Delta x}{kA}$

2nd Law: No heat engine is perfectly efficient

irreversible process: S increases

$W_{lost} = T\Delta S$

$\epsilon_{eff.} = \frac{W}{Q_h}$=work done/heat absorbed in hot reservoir

Heat engines

$W = Q_{hot} - Q_{cold}$

$\epsilon_{carn} = \frac{W}{Q_{hot}} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$

$= \frac{\Delta W}{\Delta Q}$ in a heat cycle

5 Statistical Physics

mean free path: $l == \frac{1}{\sigma n} = \frac{L}{n\pi d^2 L} = \frac{1}{n\pi d^2}$ in a volume where $n = N/V$

collision rate $= \frac{v_{rms}}{l}$

Distributions

Boltzmann: $N(E) = N_0 e^{-E/kT}$

Bose-Einstein: $F_{BE} = \frac{1}{e^{E/kT}-1}$

Fermi-Dirac: $F_{FD} = \frac{1}{e^{(E-\mu)/kT}+1}$

$\mu(T=0) = E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{\pi n_F}{L}\right)^2$

Maxwell velocity (3d) $P(v) = Cv^2 e^{-m(v-v_0)^2/2kT}$

where $C = 4\pi(m/2\pi kT)^{3/2}$

Maxwell velocity: $P(v)dv = \left(\frac{m}{2\pi kT}\right) e^{-\frac{1}{2}mv^2/kT} v^{n-1} dv$

n=1 (1D); n=3 (3D) and multiply by 4π

$v_{rms} = (3kT/m)^{1/2}$, $v_{mp} = (2kT/m)^{1/2}$, $v_{mean} = (8kT/\pi m)^{1/2}$

Planck Derivation

$$E_n = n\hbar\omega$$

$$Z = \sum_{n=0}^{\infty} e^{-n\hbar\omega/kT} = \sum x^n$$

because $x = \hbar\omega/kT < 1$ $Z = \frac{1}{1-x} = \frac{1}{1-e^{-n\hbar\omega/kT}}$

$$P(n) = Z^{-1} e^{-n\hbar\omega/kT}$$

Now, the number of oscillators:

$$< n >= \sum nP(n) = Z^{-1}\sum_{n=0}^{\infty} ne^{-n\hbar\omega/kT} = Z^{-1}\sum ne^{-ny} = -Z^{-1}\frac{d}{dy}\sum e^{-ny} = -Z^{-1}\frac{d}{dy}\left(\frac{1}{1-e^{-y}}\right)$$

where $y = \hbar\omega/kT$.

$$< n >= Z^{-1}\frac{e^{-y}}{(1-e^{-y})^2} = \frac{e^{-y}}{1-e^{-y}} = \frac{1}{e^{\hbar\omega/kt}-1}$$

$$< E >=< n > \hbar\omega = \frac{\hbar\omega}{e^{\hbar\omega/kt}-1}$$

Diffusion

- Fick's Law: $J_n = -D\nabla n$ = the flux density = number of particles passing through a unit area in unit time where n is the spatial distrobution of particles
- D is the diffusivity, $D = \frac{1}{3}v_{ave}l$ where l is the mean free path
- or the Diffusion equation: $\frac{\partial f}{\partial x} = D\frac{\partial^2 f}{\partial x^2}$ where $D = \frac{l^2}{\tau} = \frac{l^2}{l/\bar{v}} = l\bar{v}$

6 Optics

- $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$, $m = \frac{-d_i}{d_o} = \frac{h_i}{h_o}$
- Thick lens (refraction) equation: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2-n_1}{r}$where s is the distance from the image to the surface, s' is the distance from the object to the surface, r is the radius of curvature, and n_1 and n_2 are the indexed of refraction of the outside and inside of the lens, respectively.
- Lens makers equation: $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$where r_1 and r_2 are the radii of curvature (for a double convex lens, $r_1 = +$ and $r_2 = -$).
- Magnifying (simple) power of a lens: $M = \frac{\theta}{\theta_0} = \frac{x_{np}}{f}$
- f-number: $\frac{f}{D}$
- Power $= 1/f$
- $\frac{1}{f_1} + \frac{1}{f_2} + \frac{s}{f_1 f_2} = \frac{1}{f_{tot}}$
- Snell's Law: $n_1 sin\theta_1 = n_2 sin\theta_2$

- $sin\theta_C = \frac{n_2}{n_1}$

- $n = \frac{c}{v}$

- $\lambda_1 n_1 = \lambda_2 c_2$

- Interference in thin films: For 1 phase change of 180: $\frac{2t}{\lambda'} = m$ (destructive: $2t = \lambda$) and $\frac{2t}{\lambda'} = m + \frac{1}{2}$(constructive) where $m = 0, 1, 2, ...$ For 2 phase changes (2 surfaces with increasing n - thus no phase change), the destructive and constructive are reversed.

- Double-slit: $dsin\theta = m\lambda$ where $m = 0, 1, 2, ...$ (maxima)and d is the distance between the slits. For a grating: $d = m/line$.

- Single-slit: $asin\theta = m\lambda$ where $m = 1, 2, 3, ...$ (minima) and a is the width of the slit.

- For double slit interference: $\phi = \frac{2\pi}{\lambda} asin\theta$ is the difference in phase between rays from the top and bottom of each slit. $\delta = \frac{2\pi}{\lambda} dsin\theta$ is the phase difference between rays from the centers of the two adjacent slits. $I_{av} = I_0 cos^2 \frac{\phi}{2}$and $I = 4I_0 \left(\frac{sin\frac{1}{2}\phi}{\frac{1}{2}\phi} \right)^2 cos^2 \frac{1}{2}\phi$.

- phase difference: $\delta = \frac{\Delta r}{\lambda} 2\pi$ where Δr is the path difference.

- Rayleigh criteria for resolving: $\theta_{min} = \frac{\lambda}{a}$(slits of width a) and $\theta_{min} = 1.22\frac{\lambda}{D}$ (circular aperture of diameter D)

- These are all examples of **Fraunhofer diffraction**: diffraction which is observed far from the aperture where the rays are nearly parallel.

- **Fresnel diffraction** is observed near an aperture when $\lambda \approx a$. The key point here is that the maxima occur when the path difference between rays from the center and edge of the aperture is $\frac{n\lambda}{2}$. A Fresnel lens can be created from Fresnel zones: $R_n = \sqrt{n\lambda L}$ where $L = \left(\frac{1}{h} + \frac{1}{h'} \right)^{-1}$ and h and h' are the distances from the observation point and source to the center of the aperture respectively. **Fresnel Lens** (zone plate): $f = L = \frac{R_1^2}{\lambda}$. The amplitude at f equals **twice** the amplitude at $z = 0$. **Intensity equals amplitude squared** or 4 times the amplitude at $z = 0$. If the center is obstructed, R1 begins at the edge of the obstacle. Partial derivation:

  1. $b - z = \frac{n\lambda}{2}$: edges. ($z$ is the distance from the center of the aperture to the point in question (z-axis), b is the hyptenous distance

2. $b^2 = a^2 + z^2$ where a is the radius the imaginary aperture

3. $b^2 = (z + \frac{n\lambda}{2})^2 = z^2 + zn\lambda + \frac{n^2\lambda^2}{4}$

4. These give $a^2 \approx nz\lambda$ so $R_n = \sqrt{nz\lambda}$

5. max(min) = odd(even) number of zones

- Law of cosines: $c^2 = a^2 + b^2 + 2abcos\theta_c$

- light: 400nm (violed) - 700nm (red)

- Polarization: Malus's Law: $I = I_0 cos^2\theta$ (if polarized, divide by 2 if not)

- Polarization by reflection: Brewster's law: $tan\theta_p = \frac{n_2}{n_1}$The reflected light is completely polarized when incident at the angle θ_p but the refracted light is only partially polarized. If the incident light is polarized with E in the plane of incidence, there is no reflected ray.

7 Quantum Mechanics

Modern

- Photoelectric effect: $K_{max} = h\nu - \phi$ or $E = h\nu = K + \phi$ where K is the kinetic energy of the released electron and ϕ is the work function.

- Compton scattering: $\lambda' - \lambda = \frac{h}{mc}(1 - cos\theta)$

- Atomic Spectra: $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ where R is the Rydberg constant and $n_i = n_f + 1, n_f + 2, ...$

- Bohr model:

$$n\hbar = mvr$$

$$<T>= \frac{n+1}{2} <V>= -\frac{1}{2} <V>$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r}$$

$$r_n = \frac{n^2\hbar^2}{Ze^2m^24\pi\epsilon_0}$$

$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_0}\frac{1}{n^2} = -\frac{13.6eV}{n^2}$$

- De Broglie: $\lambda = \frac{h}{p}$
- Heisenberg: $\Delta x \Delta p \geq \frac{\hbar}{2}$ and $\Delta E \Delta t \geq \frac{\hbar}{2}$
- X-ray spectra: $E = -(Z-1)^2(13.6eV)$ (sheilding effect) ?
- Cherenkov radiation: the result of a particle moving faster than the speed of light in a medium: $u > \frac{c}{n}$. What is known as a Cherenkov cone is the result. Derived from Huygen's principle you make a right triangle where the hypotenuse (directed along the particle's path) is given by $u\Delta t$ and the side adjacent to the forward looking angle ϕ is given by $\frac{c}{n}\Delta t$. This angle is found to be $cos\phi = \frac{c}{un}$and the Cherenkov angle (the backward looking, opposite angle) is given by $sin\theta_c = \frac{c}{un}$.

General QM

- Schroedinger Eq.: $H\psi = \left(\frac{p^2}{2m} + V\right) = E\psi$ where $p = -i\hbar\nabla$
- Particle in a 1D box: $E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$ and $\psi_n(x) = \sqrt{\frac{2}{L}} sin(\frac{n\pi}{L}x)$ (odd parity, n=1,2,3,...). For odd parity: use cosine and $n \to 2n+1$ with $n = 0, 1, 2, ...$
- Scattering and bound states:

  1. step problems: solutions of form e^{ikx}(if E+V positive), $e^{\kappa x}$(if E+V negative), or sines and cosines (dip).
  2. penetration depth: $\delta = \frac{1}{\kappa} = \frac{\hbar}{\sqrt{2m(V-E)}}$
  3. δ-function potential: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \alpha\delta(x)\psi = E\psi$ and $\Delta\left(\frac{d\psi}{dx}\right) = -\frac{2m\alpha}{\hbar^2}\psi(0)$

- eigenvalues: $\vec{L}^2 = l(l+1)\hbar^2$, $L_z = m\hbar$, and the same from $\vec{S}$ and $\vec{J}$.
- $\vec{J}^2 = (\vec{L}^2 + \vec{S}^2)^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L}\cdot\vec{S}$
- $E_{rot} = \frac{\vec{L}^2}{2I} = \frac{l(l+1)\hbar^2}{2I}$
- **molecule energy levels**: $E_{\nu,l} = -V_0 + (\nu + \frac{1}{2})\hbar\omega + \frac{l(l+1)\hbar^2}{2I} = potential + vibrational + rotational$
- Time evolution:
$$i\hbar\frac{dA}{dt} = [A, H] \text{ (Heisenberg)}$$
$$U = e^{-iEt/\hbar}$$
$$A^{(H)} = U^{+}A^{(S)}U$$

- Commutators:

$$[x_i, p_i] = i\hbar\delta_{ij}$$

$$[\delta_i, \delta_j] = i\epsilon_{ijk}\hbar\delta_k$$

$$\{\delta_i, \delta_j\} = \frac{1}{2}\hbar^2\delta_{ij}$$

- Fermions:

antisymmetric: $\Psi \to (-1)^{L+S+1=odd}$

symmetric: $\Psi \to (-1)^{L+S=even}$

- Pauli matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ where $\sigma_i\sigma_j = i\epsilon_{ijk}\sigma_k + \delta_{ij}I$

- Spin operators:

$$S_i = \frac{\hbar}{2}\sigma_i \text{ and } [S_i, S_j] = i\hbar S_k \text{ and } \{S_i, S_j\} = \frac{\hbar^2}{2}\delta_{ij}I$$

$$S_z = \frac{\hbar}{2}(|+><+| - |-><-|)$$

$$S_x = \frac{\hbar}{2}(|+><-| + |-><+|)$$

$$S_y = \frac{\hbar}{2}(-i|+><-| + i|-><+|)$$

$$|S_z; \pm>= |\pm>$$

$$|S_x; \pm>= \frac{1}{\sqrt{2}}(|+> \pm|->)$$

$$|S_y; \pm>= \frac{1}{\sqrt{2}}(|+> \pm i|->)$$

$$S_x^2 = S_y^2 = \frac{\hbar^2}{4}(|+><+| + |-><-|)$$

$$\vec{S} \cdot \hat{n} = \frac{\hbar}{2}\begin{pmatrix} cos\beta & e^{-i\alpha}sin\beta \\ e^{i\alpha}sin\beta & -cos\beta \end{pmatrix}$$

$$|\vec{S} \cdot \hat{n}; \pm>= cos\frac{\beta}{2}|+> \pm sin\frac{\beta}{2}e^{i\alpha}|->$$

- SHO:

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^+)$$

$$p = i\sqrt{\frac{m\hbar\omega}{2}}(-a + a^+)$$

$$a|n>= \sqrt{n}|n-1>$$

$$a^+|n+1>= \sqrt{n+1}|n+1>$$

$$[a, a^+] = 1$$

$$a|0>= 0$$

$$|n>= \left[\frac{(a^+)^n}{\sqrt{n!}}\right] = 1$$

- Particle: cross section: $\sigma = \frac{A}{n}$ and width: $\Gamma = \frac{1}{\tau}$. COM energy: $S = [(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2]^{1/2}$

8 Atomic

Selection Rules (electric dipole transitions)

one electron (energy levels): $\Delta l = \pm 1$

$\Delta j = 0, \pm 1$

L·S coupling (spectroscopic): $\Delta S = 0$

$\Delta L = 0, \pm 1$

$\Delta J = 0, \pm 1$ (not 0 to 0)

$S = |s_1 - s_2|...|s_1 + s_2|$ $\quad L = |l_1 - l_2|...|l_1 + l_2|$

$J = |L - S|...|L + S|$

Russel-Saunders notation

- notation: ${}^{2S+1}L_J$
- Spin-Orbit (SO) splitting: $\Delta E = A\vec{L} \cdot \vec{S} = \frac{A}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2) = \frac{A}{2}(J(J+1) - L(L+1) - S(S+1))$
- Level split by Zeeman/H splitting (in other words, determined by J): the energies of the splittings are $\Delta E = \mu_b B g m_j'$ where $m_j' = -J...J$ and the Lande g-factor is $g = 1 + \frac{J(J+1)+S(S+1)+L(L+1)}{2J(J+1)}$.

Magnetic moment and Spin

- magnetic moment: $\vec{\mu} = \frac{q}{2m}\vec{L} = \frac{q\hbar}{2m}$ $(\vec{L} \to \hbar)$
- $\vec{\tau} = \vec{\mu} \times \vec{B} = \frac{d\vec{L}}{dt}$
- Larmar precesion: $\omega_L = \frac{e}{2m}B$
- $H = U = -\vec{\mu} \cdot \vec{B} = \frac{e}{2m}\vec{L} \cdot \vec{B} = \frac{eB}{2m}L_z = \hbar\omega_L m_e$
- The wave functions of atomic electrons are uneffected by the application of a magnetic field.
- ADDING SPIN...
- $\mu_s = \frac{qg}{2m}\vec{S}$
- $\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_s = \frac{e}{2m}(\vec{L} + g\vec{S})$

- $< S \cdot B >= \frac{<S \cdot J><J \cdot B>}{<J^2>}$

- $< J \cdot B >= \hbar B m_j$

Electrons in same orbital: possible terms for $(nl)^k$ w/ l=0,1,2

ns	^{2}S
ns^2	^{1}S
np or np^5	^2P
np^2 or np^4	^1S, ^1D, ^3P
np^3	^2P, ^2D, ^4S
np^6	^1S
nd or nd^9	^2D
nd^2 or nd^8	^1S, ^1D, ^1G, ^3P, ^3F
nd^{10}	^1S

9 Solid State

- phonons in a crystal:
$$\begin{gathered}\text{(1D) } m\ddot{x}_n = k(x_{n+1} - x_n) + k(x_{n-1} - x_n) \\ x_n = x_0 e^{i(knl - \omega t)} \text{ (discretize x)} \\ p = \hbar k\end{gathered}$$

- Fermi derivation:
$$\begin{gathered}n_p = \frac{1}{e^{(E-\mu)/kT}+1} \text{(F-D)} \\ \sum_p = \frac{2V}{(2\pi\hbar)^3} \int d^3p = \frac{2V}{(2\pi\hbar)^3} 4\pi \int p^2 dp \\ N = \frac{V}{\pi^2\hbar^3} \int_0^{p_F} p^2 dp \Rightarrow p_F = (3\pi)^2 \hbar (N/V)^{1/3} \\ \epsilon_F = \frac{p_F^2}{2m} = (3\pi^2)^{2/3} \frac{\hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3} \\ E = \frac{3}{5}\epsilon_F N\end{gathered}$$

10 Other

- Error propogation:
$$\begin{gathered}\text{systematic: } du = \left(\frac{\partial u}{\partial x}\right)_{y,z} dx + \left(\frac{\partial u}{\partial y}\right)_{x,z} dy + \left(\frac{\partial u}{\partial z}\right)_{x,y} dz \\ \text{random: } (du)^2 = \left(\frac{\partial u}{\partial x}\right)^2_{y,z} (dx)^2 + \left(\frac{\partial u}{\partial y}\right)^2_{x,z} (dy)^2 + \left(\frac{\partial u}{\partial z}\right)^2_{x,y} (dz)^2 \\ \text{where we equate } du \text{ with the standard deviation } \sigma_u\end{gathered}$$

If the variables are correlated, you must include cross terms.

11 Units

- $velocity = \sqrt{energy/mass}$
- $pressure = force/area$
- $h = energy \times time$
- $e^2 = energy \times distance$
- $energy = mass \times velocity^2$
- $h/e^2 = time/distance$
- $h/m = energy \times time/mass = distance^2/time$
- Intensity ($\vec{S}$) (power/unit area = W/m^2)
- Power: (J/s $\equiv$N·m/s)
- specific heat: c (J/kg·K)
- entropy: (J/K)
- current: A (V/Ω)
- capacitance: F (C/V)
- inductance: H (Wb/A = T·m^2/A)
- conductivity: ($1/\Omega \cdot m$)
- ressistivity: ($\Omega \cdot m$)
- magnetic flux: Wb (T·m^2)

12 Constants

- $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4

13 Math

- Gaussian: $\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\bar{x})^2}{2\sigma}}$
- Poisson: $\frac{\mu^r e^{-\mu}}{r!}$
- Cylindrical coordinates: $dA = rdrd\phi$ and $dV = rdrd\phi dz$
- Spherical coordinates: $dA = r^2 sin\theta d\theta d\phi$ and $dV = r^2 sin\theta drd\theta d\phi$
- $cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ and $sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$
- $\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$
- $\int_0^\infty e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$
- $\int_0^\infty xe^{-\alpha x^2} = \frac{1}{2\alpha}$
- $\int_0^\infty x^2 e^{-\alpha x^2} = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$
- $f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + ...$ or in 3D: $f(\vec{x}_0 + \vec{x}) = f(\vec{x}_0) + \vec{x} \cdot \nabla f(\vec{x}_0) + \frac{1}{2!}(\vec{x}_0 \cdot \nabla)(\vec{x}_0 \cdot \nabla)f(\vec{x}_0) + ...$
- $e^x = 1 + x + \frac{x^2}{2!} + ...$