Signatures of Silicon

Spin and valley states in SiGe

2DEG spin relaxation in Si quantum wells

ESR: Electron spin resonance on macroscopic samples provides a key and available measure of spin coherence properties in silicon quantum well heterostructures, though not a one-to-one correspondence.

D'yakonov and Perel' spin relaxation: Rashba spin orbit coupling produces an effective magnetic field in the plane of the quantum well. Scattering causes a random switching of this field, dominating the spin dynamics for most quantum wells. We solve the spin density matrix master equation as a function of static magnetic field direction to get the spin relaxation times T_1 and T_2.

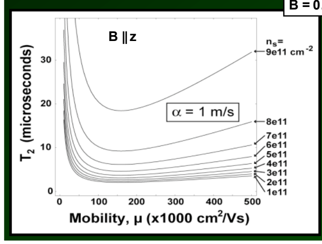
$$\frac{d\rho}{dt} = \frac{i}{\hbar}[\rho, H]$$

$$H = -g\mu_B B\hat{n}\cdot\vec{\sigma}/2 + H_R(t)$$

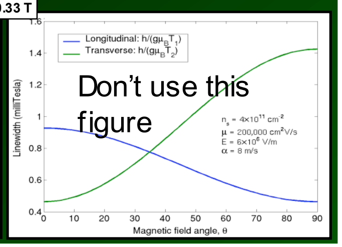
$$\omega_L = \frac{eB}{m} \qquad \tau_p \approx \frac{m^*\mu}{e}$$

$$\frac{1}{T_1(\theta)} = \left[\frac{2\alpha^2 p_F^2\tau_p}{1+(\omega_L\tau_p)^2}(\cos^2\theta+1)\right]/\hbar^2$$

$$\frac{1}{T_2(\theta)} = \left[2\alpha^2 p_F^2\tau_p\sin^2\theta + \frac{\alpha^2 p_F^2\tau_p}{1+(\omega_L\tau_p)^2}(\cos^2\theta+1)\right]/\hbar^2$$

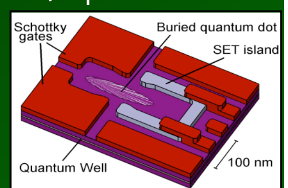


B = 0.33 T



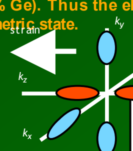
Quantum dot qubit spin-flip time, T_1

Spin relaxation across Zeeman sublevels involves emission of a phonon and thus transfer of energy to the environment. These transition rates are usually calculated with the Golden Rule.



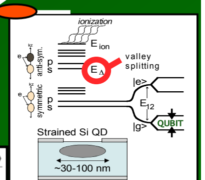
P:Si: Donors in silicon provide the closest physical analogue to Si QD localized states and a natural historical precedent. Roth/Hasegawa developed a very successful theory for T_1 based on spin mixing with nearby 1s-like states as well as with nearby conduction bands.

For [001] compressively strained silicon (as in a QW), the four plane like valleys move up in energy relative to the *z* valleys (~150meV for 30% Ge). Thus the electron exists only in the +/- *z* valleys in either a symmetric or anti-symmetric state.



This valley splitting eliminates the primary Roth/Hasegawa mechanism.

QD: Due to Rashba SOC and the much closer spacing of dot levels, a new mechanism of relaxation emerges: a direct phonon process across higher dot levels (p-like and up) mediated by SOC.



$$\frac{1}{T_1} \approx \frac{(m^*\alpha)^2}{210\pi\hbar}\left(\frac{g\mu_B B}{\hbar}\right)^7\left(\frac{35\Xi_d^2+14\Xi_d\Xi_u+3\Xi_u^2}{v_l^7}+\frac{4\Xi_u^2}{v_t^7}\right)\left[(\Sigma_{xx}+\Sigma_{yy})(3+\cos 2\theta)+(\Sigma_{xx}-\Sigma_{yy})\cos 2\phi\sin^2\theta\right]$$

$$\Sigma_{xx} = \sum_r \frac{\langle n|x|r\rangle\langle r|x|n\rangle}{E_{nr}}$$

Contrary to the Roth/Hasegawa theory, we predict for QDs a relaxation rate proportional to the seventh power of the magnetic field (versus to the fifth) and a rate very dependant on the structure of the dot (versus independent).

Estimate for E_{eg}=0.1meV dot: ($B=B_z$, T=0 K)

$$\frac{1}{T_1} \sim 736\ B^7\ s^{-1}$$

Electron dynamics in silicon quantum well heterostructures are heavily influenced by Rashba spin-orbit coupling and a many-valley silicon conduction band.

Spin-Orbit Coupling

Bulk Si: Spin-orbit (SO) coupling in silicon is weak, the asymmetric g-factor of a conduction electron being very nearly 2: $\Delta g_{\parallel} = -0.003$ $\Delta g_{\perp} = -0.004$

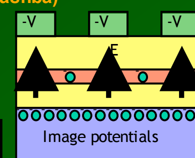
The SO term can be derived from the Dirac equation for a relativistic electron. $H_{SO} \propto \vec{\nabla}V\cdot(\vec{\sigma}\times\mathbf{p})$

Si 2D Heterostructure

A strong electrostatic potential, either due to charge separation in the device or to an external electric field, can contribute significantly to SO coupling.

Structural Inversion Asymmetry (Rashba)

A large field in the z-direction (roughly 10^6 V/m) due to a donor layer (2DEG) or gate/image potentials (WiscQC) generates the familiar Rashba SO Hamiltonian.



$$H_R = \alpha(p_x\sigma_y - p_y\sigma_x)$$

$$\alpha = 8.4\ \text{m/s}\ (Si_{0.75}Ge_{0.25},\ n_s = 10^{11}\text{cm}^{-2})$$

Wilamowski, *et. al.* PRB

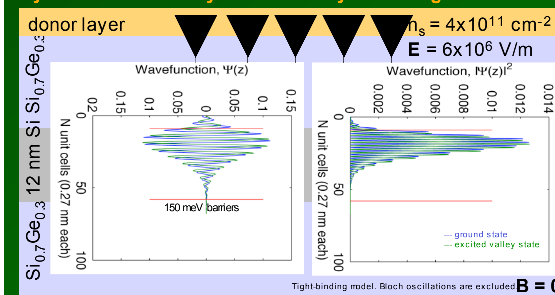
$$\alpha(E>>0) \approx \frac{\hbar}{2m^*}\frac{\Delta^{SO}}{E_g^d}\frac{2E_g^d+\Delta^{SO}}{(E_g^d+\Delta^{SO})(2E_g^d+2\Delta^{SO})}eE$$

de Andrada e Silva, *et. al.* PRB

Valley states in silicon quantum dots

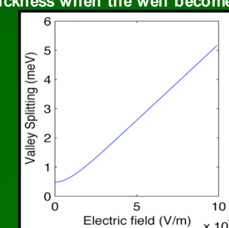
A single electron trapped in strained silicon exists as a linear combination of the two conduction band minima (or valleys) along the k_z axis.

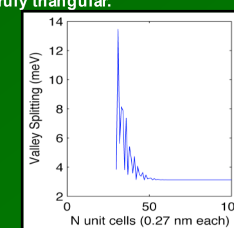
Symmetric and anti-symmetric valley electron ground states



Boykin, *et. al.* cond-mat/0309663

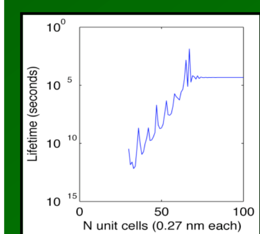
Valley splitting: The sharp potential from the quantum well and electric field splits the degeneracy of the valley states. The valley splitting oscillates with monolayer number up to some critical thickness when the well becomes truly triangular.

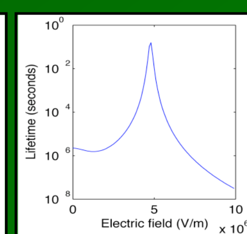


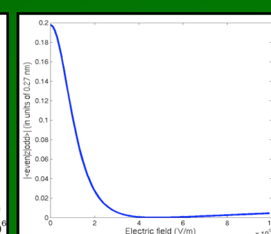


Long valley-state lifetimes

At low temperatures in the electric dipole approximation, single electron state lifetimes are determined by the matrix element connecting the even and odd valley states and the valley splitting between them. Despite large energy splitting, the smallness of the connecting matrix element can result in long excited state lifetimes. $1/\tau_v \sim E_v^5|\langle even|z|odd\rangle|^2$







Experimental implications

- Quantum dot transport measurements should reveal valley signatures just as they do spin states.
- The optical transition speed is quite slow across valley states.
- Signal changes from the presence of valley states may be observable in electrically detected resonance experiments.
- Shubnikov de-Haas oscillations already reveal valley splitting in many-particle quantum wells.
- Leakage into excited valley states may have implications for silicon quantum computing.

References:

Tahan, et. al., *Decoherence of electron spin qubits in Si-based quantum computers*, PRB

Friesen, Tahan, et. al., *Spin readout and initialization in a semiconductor quantum dot*, PRL

Tahan, et. al., *Spin relaxation in SiGe two dimensional electron gases*, cond-mat

Tahan, et. al., *Spin and orbital relaxation in silicon quantum well quantum dots*,

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