Spin-based quantum dot quantum computing: the ultimate in quantum electronics

qc.physics.wisc.edu

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ECE746-Quantum Electronics, Guest Lecture
1:00pm, Oct. 9, 2003

ARDA

UW-Madison Solid-State Quantum Computing

Mark Eriksson (Physics)
Robert Blick (ECE)
Sue Coppersmith (Physics)
Robert Joynt (Physics)
Max Lagally (Materials Science)
Dan van der Weide (ECE)
Mark Friesen (Materials Science and Physics)
Don Savage (Materials Science)
Levente Klein (Physics)
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Keith Slinker (Physics)
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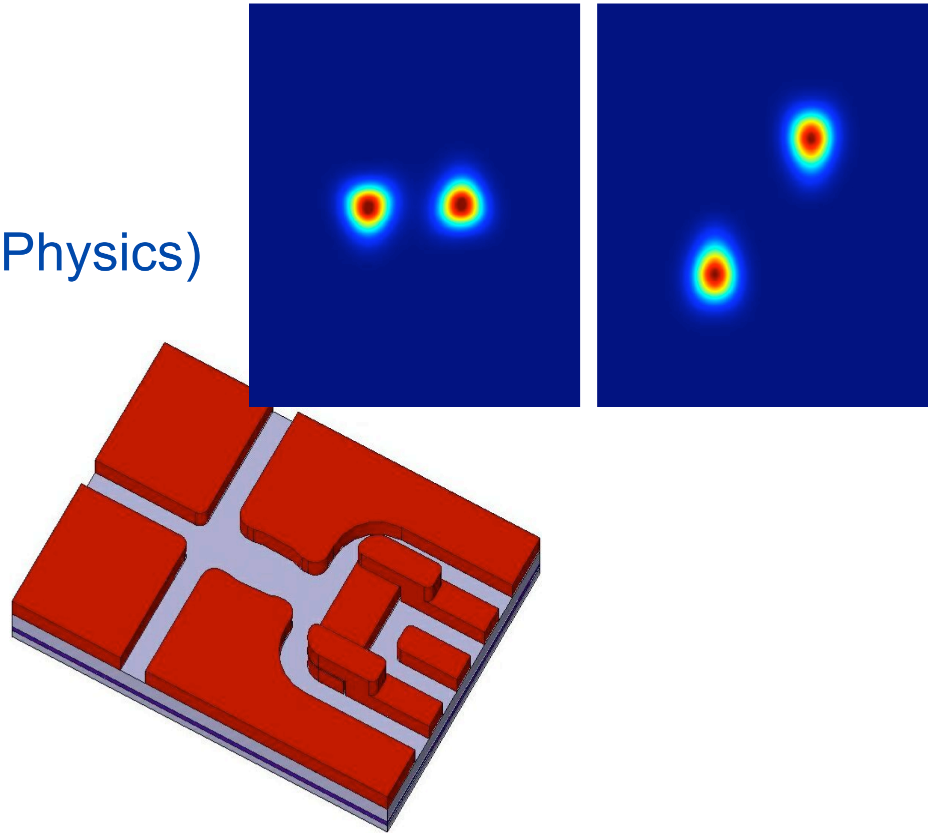
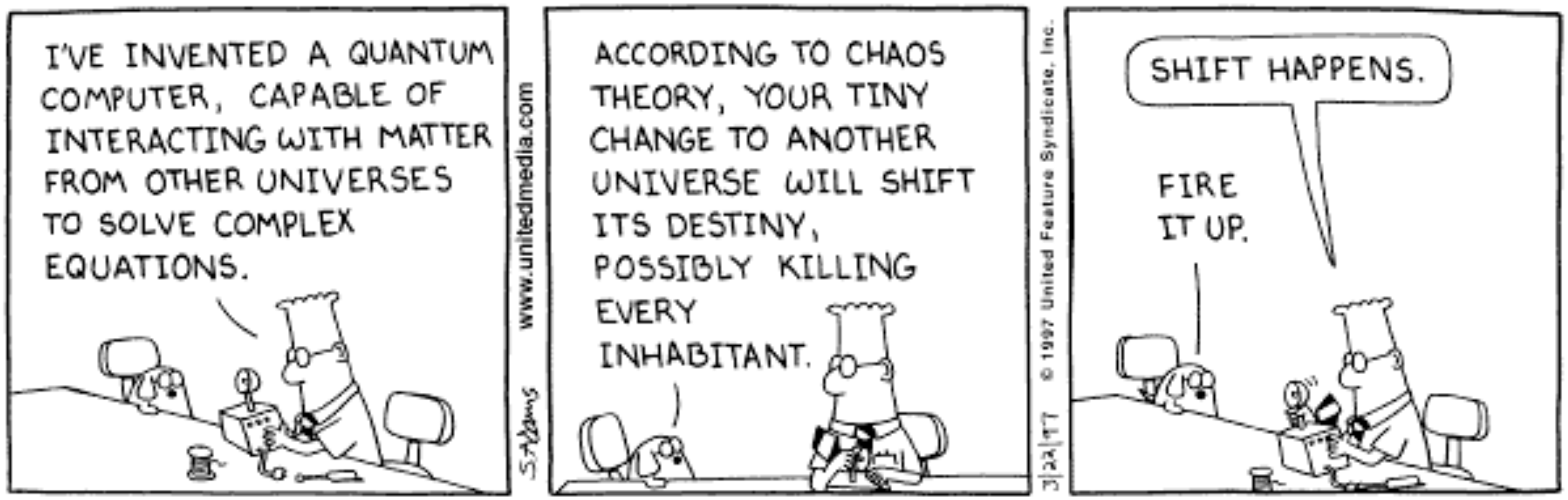
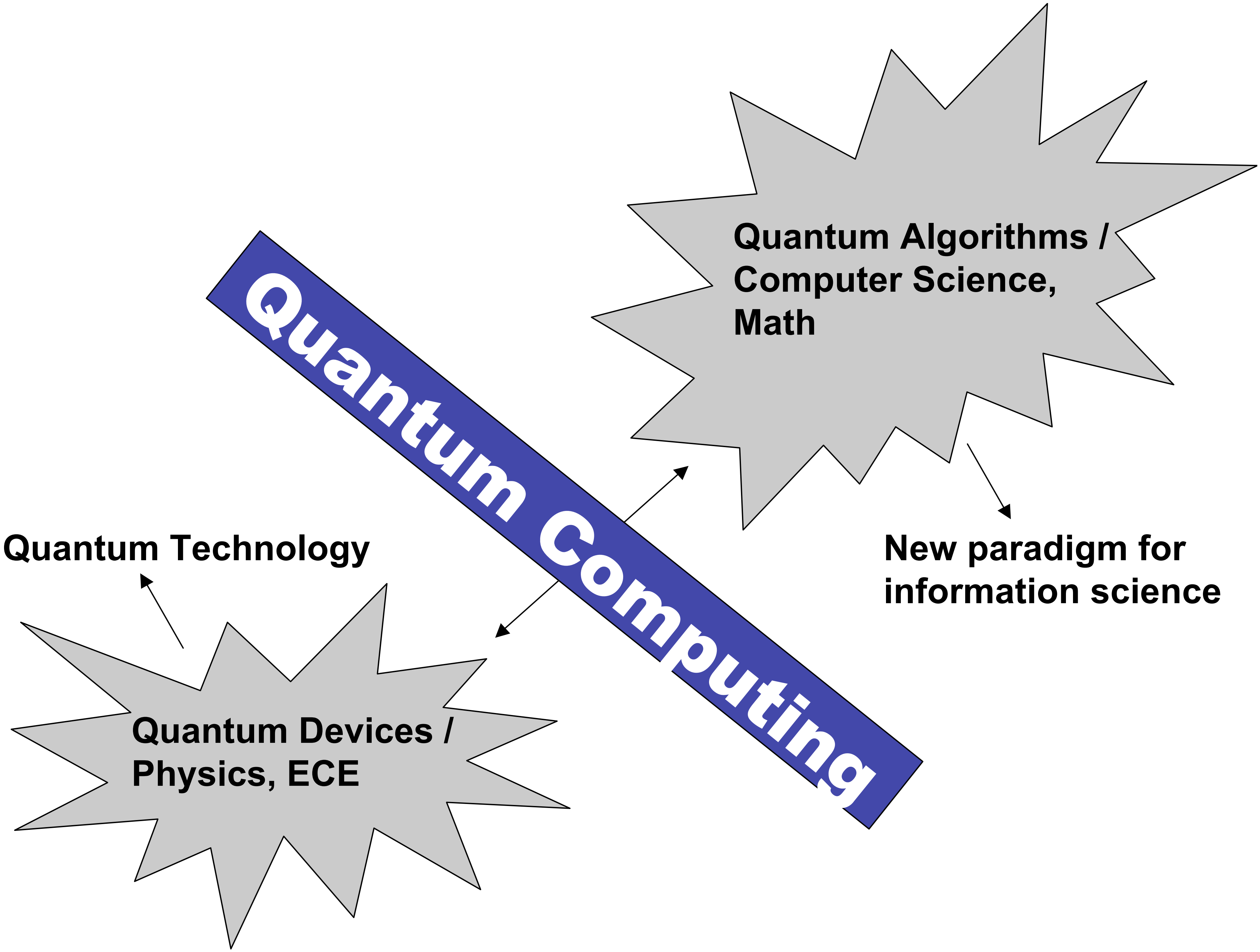
Jim Truitt (ECE)
Kristin Morgenstern (Physics)





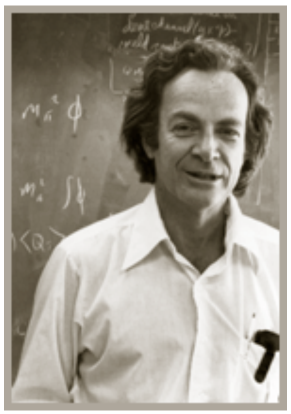
Outline

- **Quantum Computing**
- **Motivation**
- **Quantum over classical**
- **Building a QC**
- **Formalism**
- **A Good Qubit: Spin**
- **Quantum dot architectures**
- **Universal QC**
- **A quantum well quantum dot**
- **Entanglement and CNOT**
- **One qubit operations**
- **Encoded qubits**
- **Readout schemes**
- **Initialization schemes**
- **Quantum Error Correction**
- **Experimental Progress**


Quantum Algorithms / Computer Science, Math
New paradigm for information science
Quantum Computing
Quantum Technology
Quantum Devices / Physics, ECE

Motivation

Shor Algorithm for prime factorization: killer application

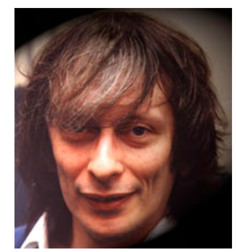


R. Feynman

Peter Shor

Charles Bennett

David Deutsch



Error correction on a QC is possible

Quantum over classical

- Superposition - Parallelism
- Interference
- Entanglement

Building a QC

- Good, scalable qubit
- Two-qubit entanglement operation
- Fast readout/measurement of qubit
- Fast initialization / source of new qubits
- Quantum error correction
- Flying qubits

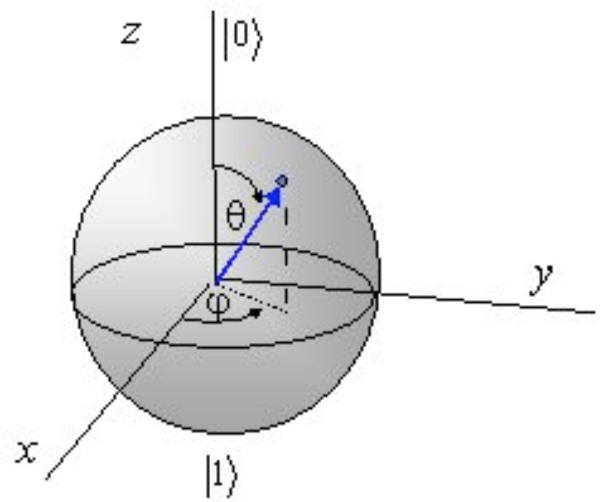
Formalism

Quantum superposition

Qubit: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\longrightarrow$ $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}$

"off" **"on"** **"off AND on"**

The Bloch sphere



$$w = w_0|0\rangle + w_1|1\rangle \equiv \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Multiple qubits:

$$|0\rangle \otimes |1\rangle \otimes |0\rangle =$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} =$$

$$\begin{pmatrix}1\begin{pmatrix}0\\1\end{pmatrix}\\ 0\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = [8 \times 1]$$

dimensional Hilbert space

Formalism

State vector formalism of quantum mechanics

$$H|\psi\rangle = E|\psi\rangle$$

Density matrix formalism

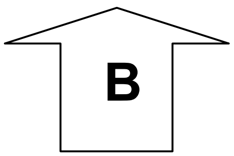
$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] \qquad \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

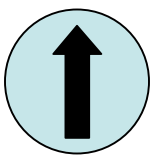
$$\rho_{|0\rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \rho_{|1\rangle} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \rho_{|+\rangle} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\rho_{QC} = \rho_1 \otimes \rho_2 \otimes \rho_3 \otimes \cdots$$

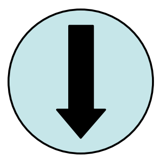
Good qubit: Spin

- Electronic or nuclear spin ½
- Natural 2 level system
- Long coherence times
- Scalable (?) in semiconductor structures

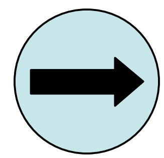




$$\rho_{|0\rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



$$\rho_{|1\rangle} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

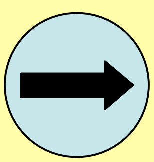


$$\rho_{|+\rangle} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Example:

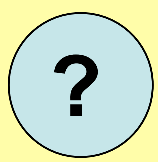
$T_1 >> T_2$

$t = 0$



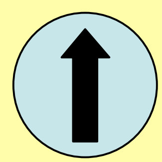
$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$T_1 > t > T_2$



$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

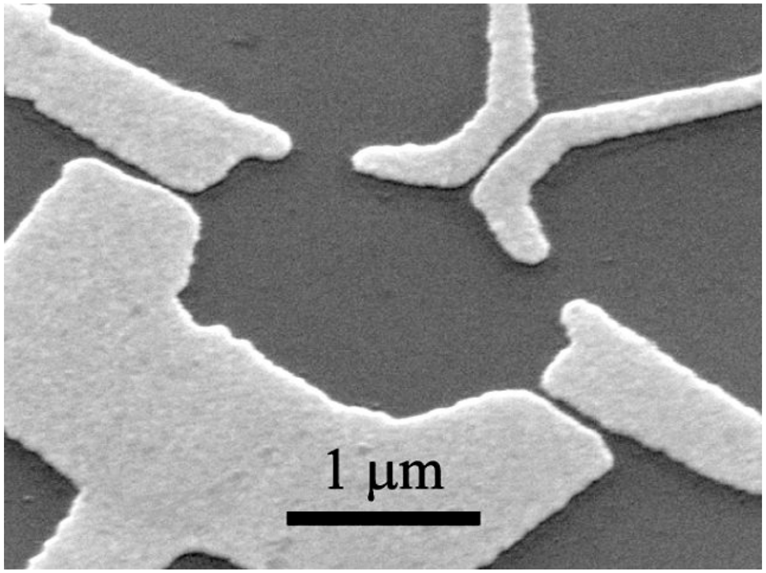
$t > T_1$



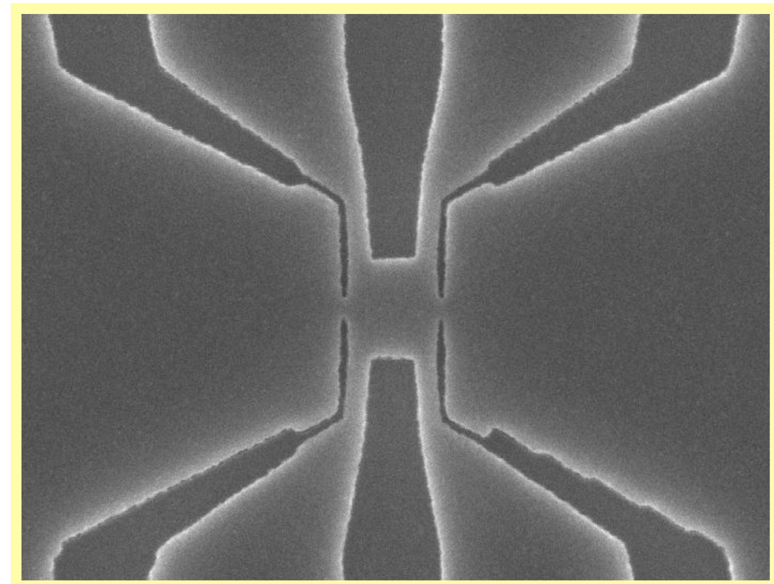
$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Quantum Dot Architectures

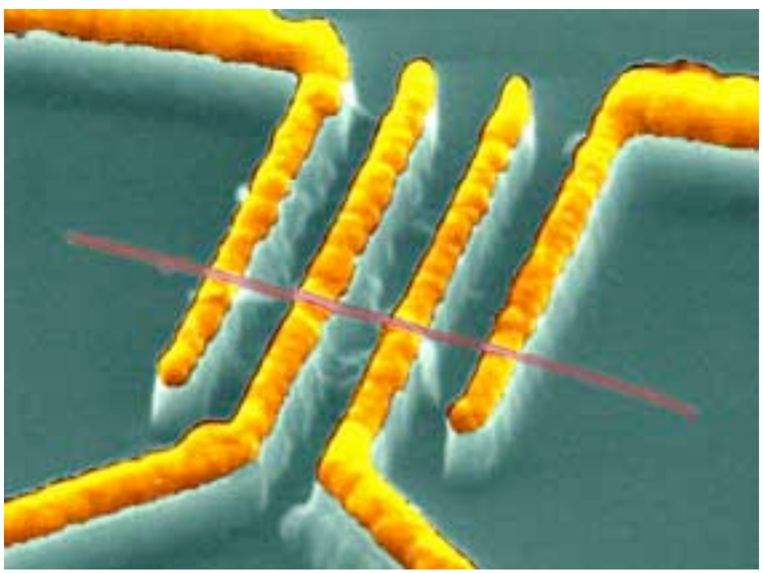
GaAs/
AlGaAs



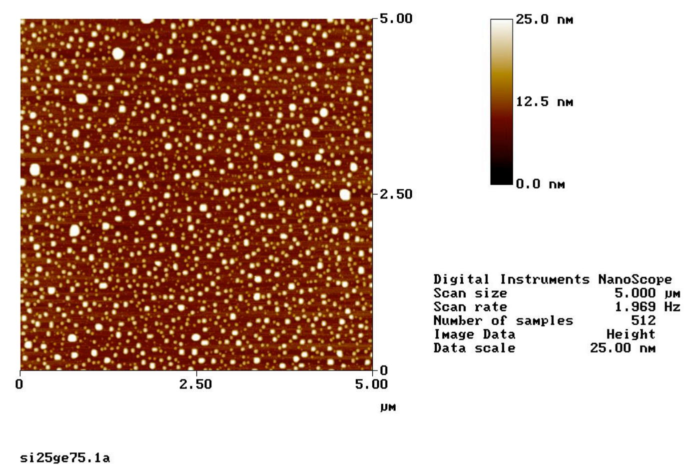
Si/SiGe



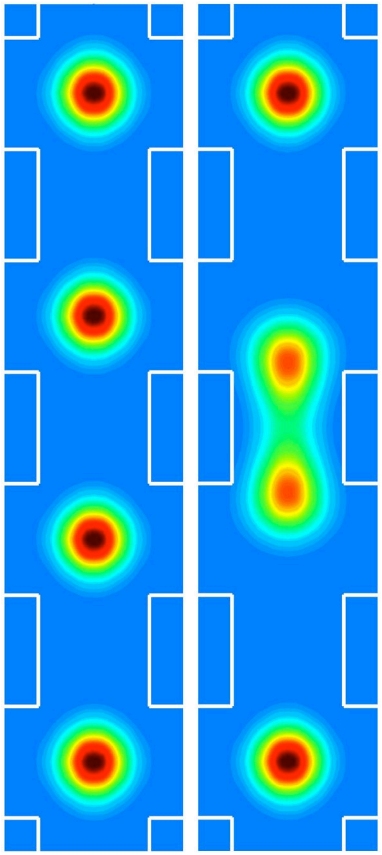
Carbon Nanotubes



Ge Huts

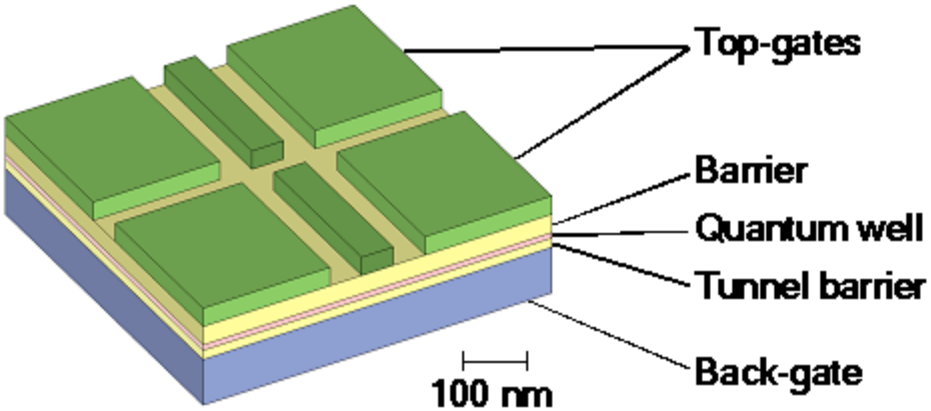


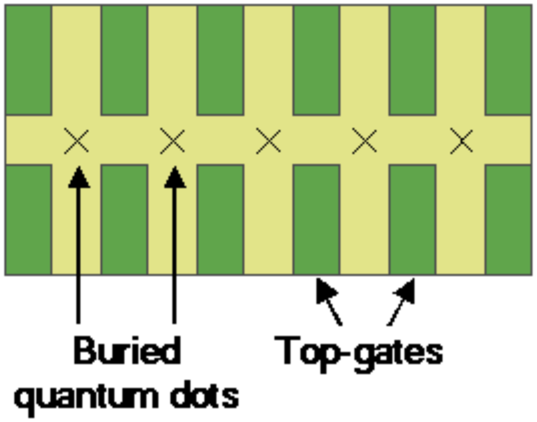
Wisconsin QDQC design



1. **Gated Quantum Dot QC** (Loss & DiVincenzo)
   - 1 electron spin = 1 qubit
   - Self aligning to gates (no need to align to donors)
   - Fast operations through *Heisenberg exchange*
   - Scalable (hopefully)
2. **Silicon**
   - Long decoherence times (T_2 ~ milliseconds for P:^{28}Si)
   - Low spin-orbit coupling
   - Spin-zero nuclei ^{28}Si
3. **Back-gate**
   - Size-independent loading and well-screened manipulation of dots

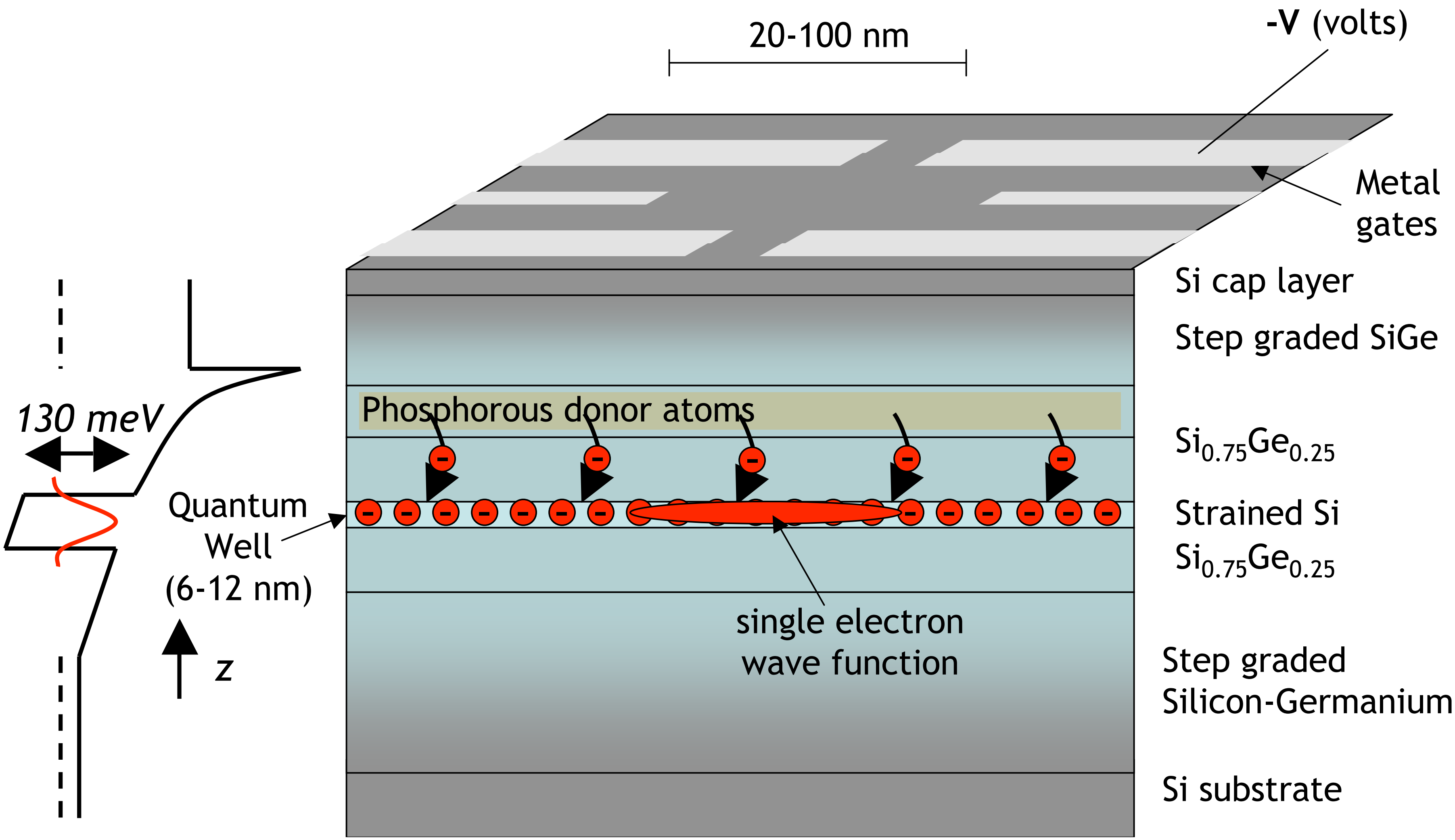
[Freisen, *et.al.*, APL]
[Freisen, *et.al.*, PRB]





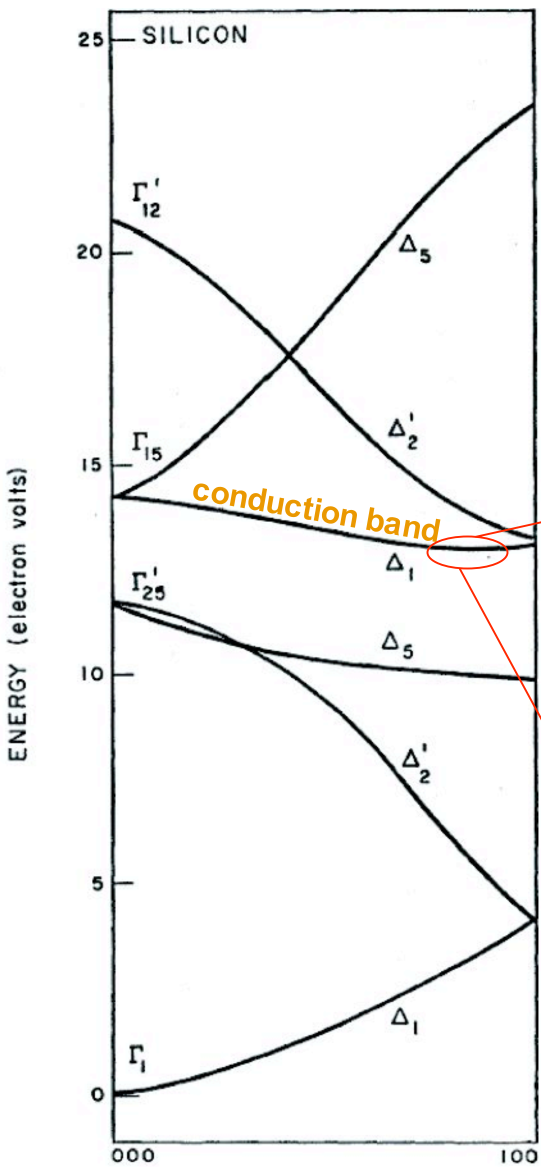
A quantum well quantum dot

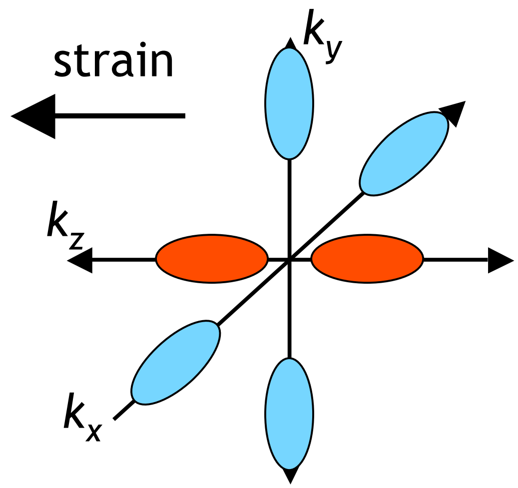
Goal: **a single electron tunably confined vertically and horizontally in a semiconductor nanostructure**

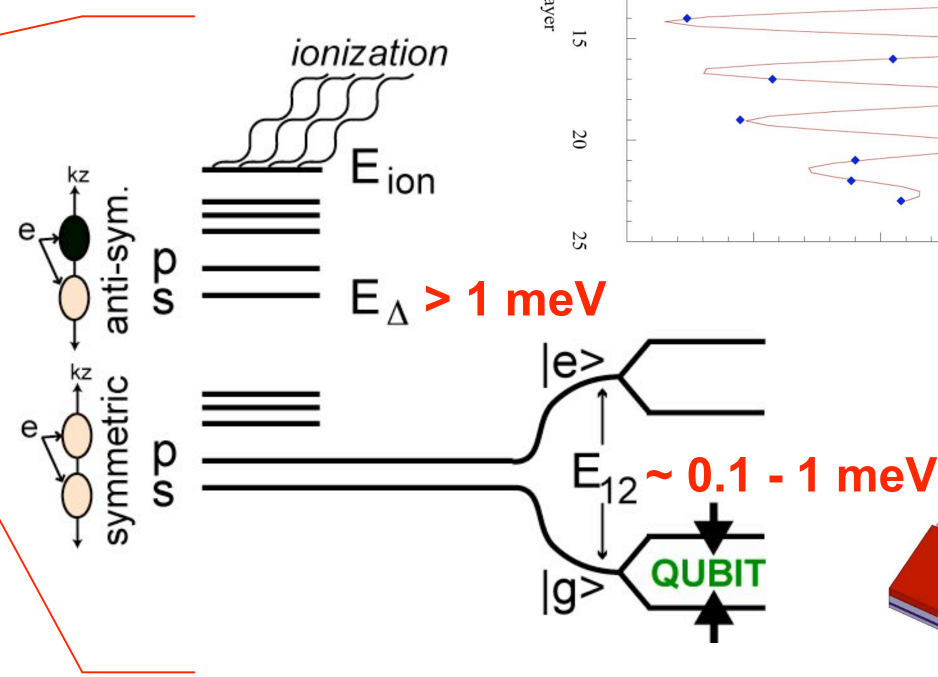


Details...

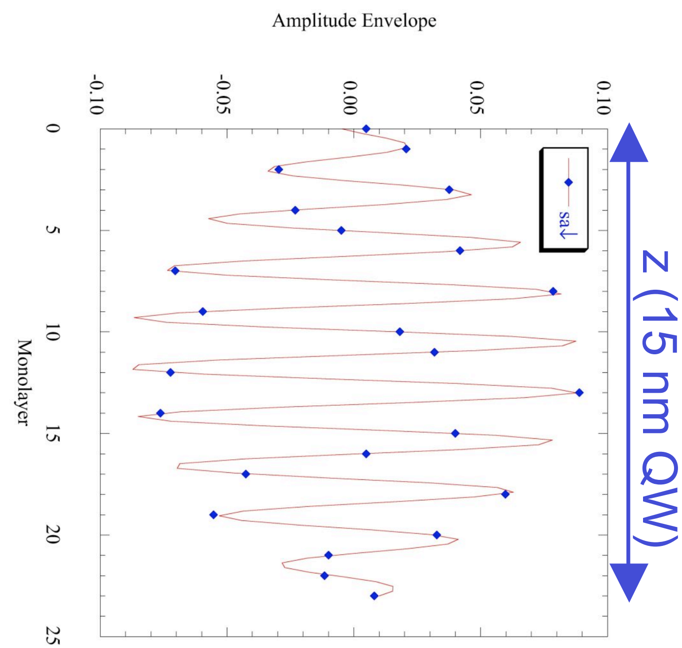
$$|\Psi\rangle = \text{Envelope} \times \text{Bloch}$$

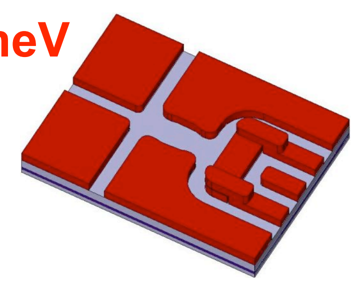




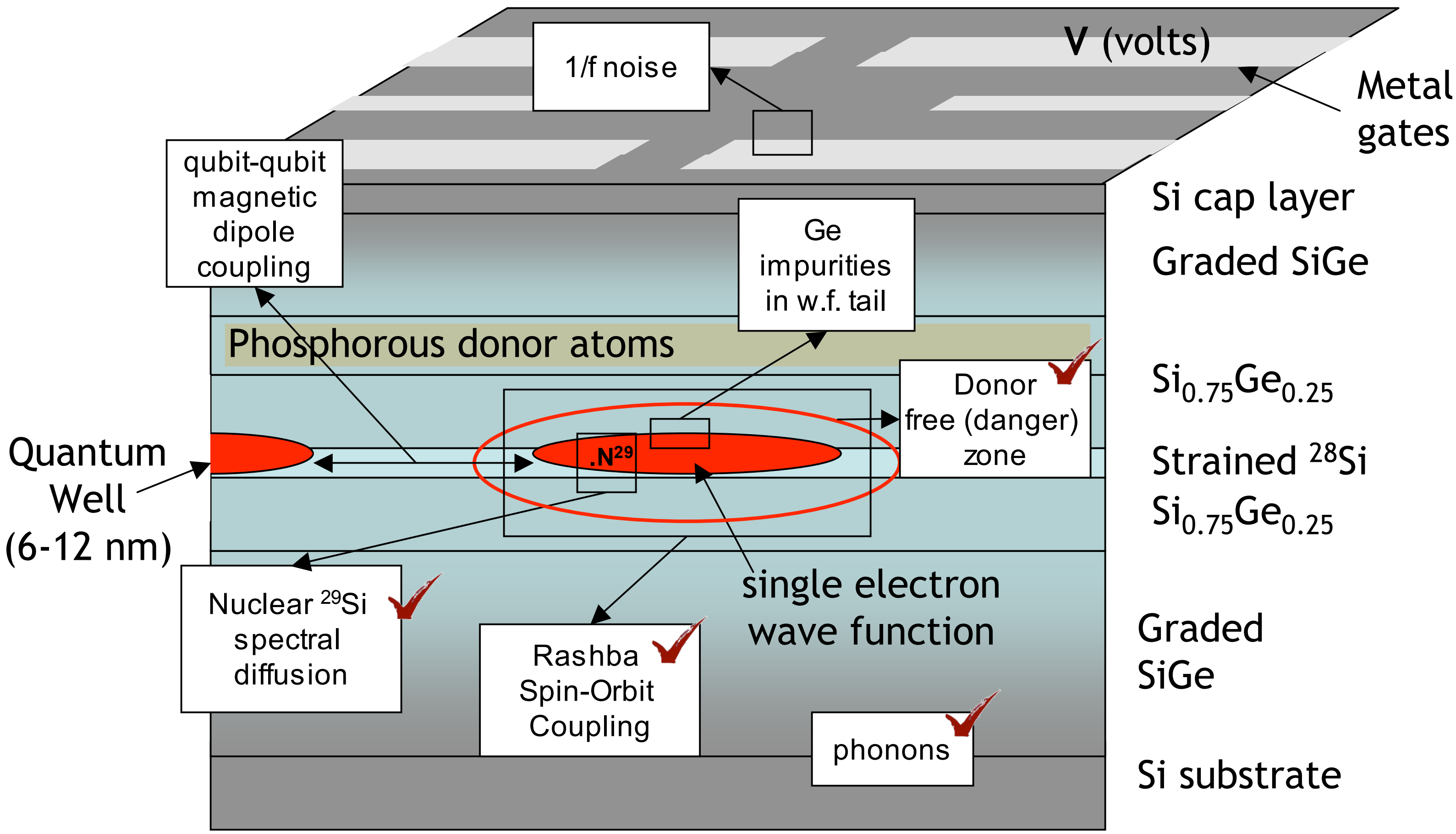


Bloch functions





Decoherence

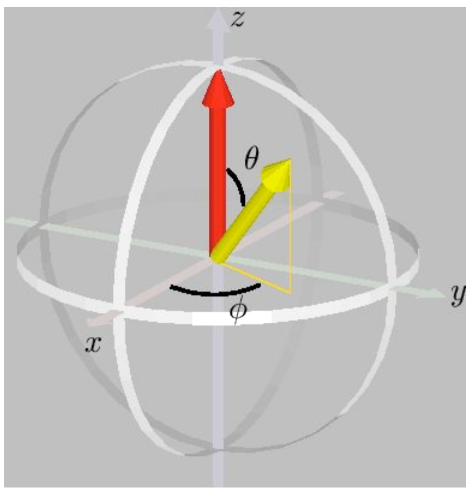


Universal QC

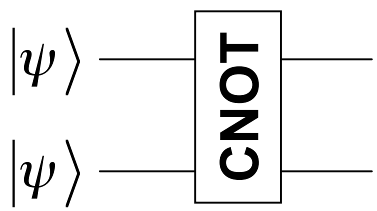
Quantum Algorithms / Computer Science, Math

A *universal* set of gates can compute an arbitrary function (e.g. NAND for classical computation)

Single qubit gates and CNOT are a universal set of gates for quantum computation.



$$\begin{aligned} \mathrm{cnot}|00\rangle &= |00\rangle \\ \mathrm{cnot}|01\rangle &= |01\rangle \\ \mathrm{cnot}|10\rangle &= |11\rangle \\ \mathrm{cnot}|11\rangle &= |10\rangle. \end{aligned}$$



$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

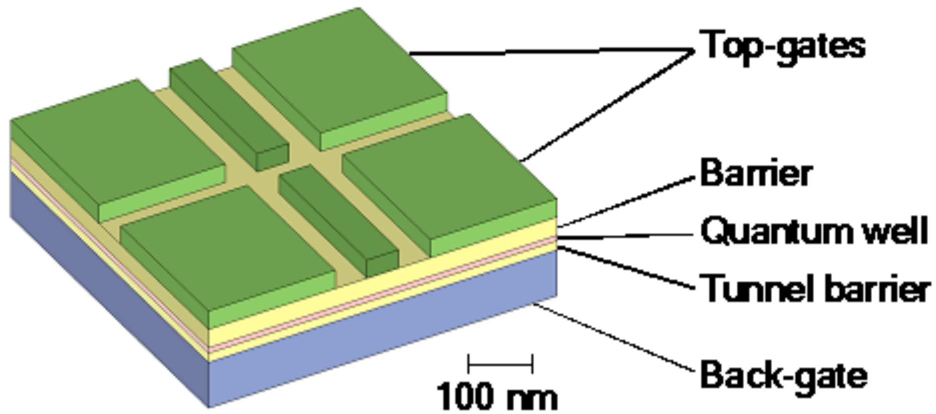
Entanglement

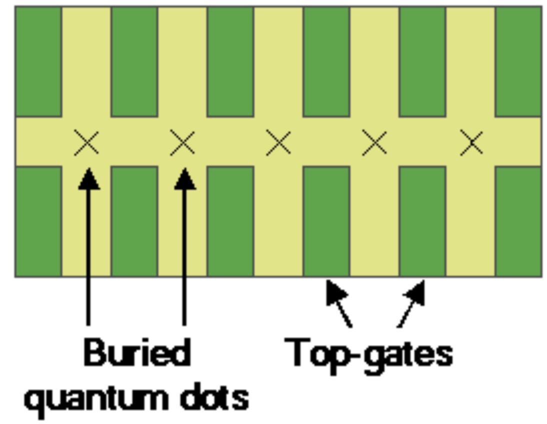
A quantum state of N qubits that cannot be written as a N-tensor product is said to be entangled.

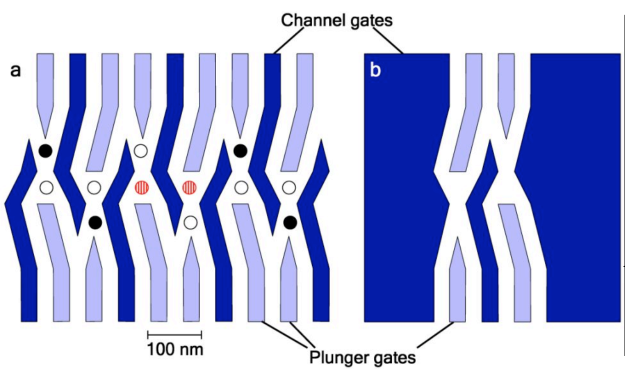
$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \neq |?\rangle_1 \otimes |?\rangle_2$$

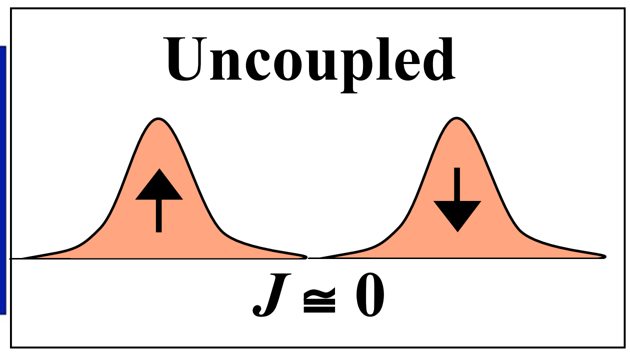
"Spooky action at a distance"

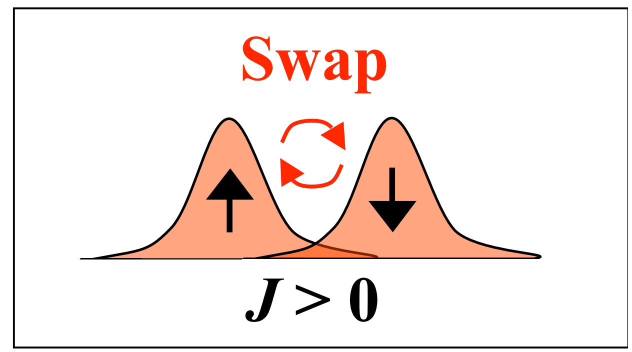
Exchange and CNOT











$$H_{2\ \text{quantum dots}} \rightarrow H_{\text{eff}} = J\ \boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

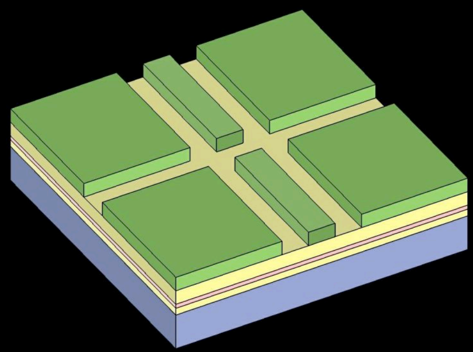
SWAP doesn't entangle but Sqrt[SWAP] does.

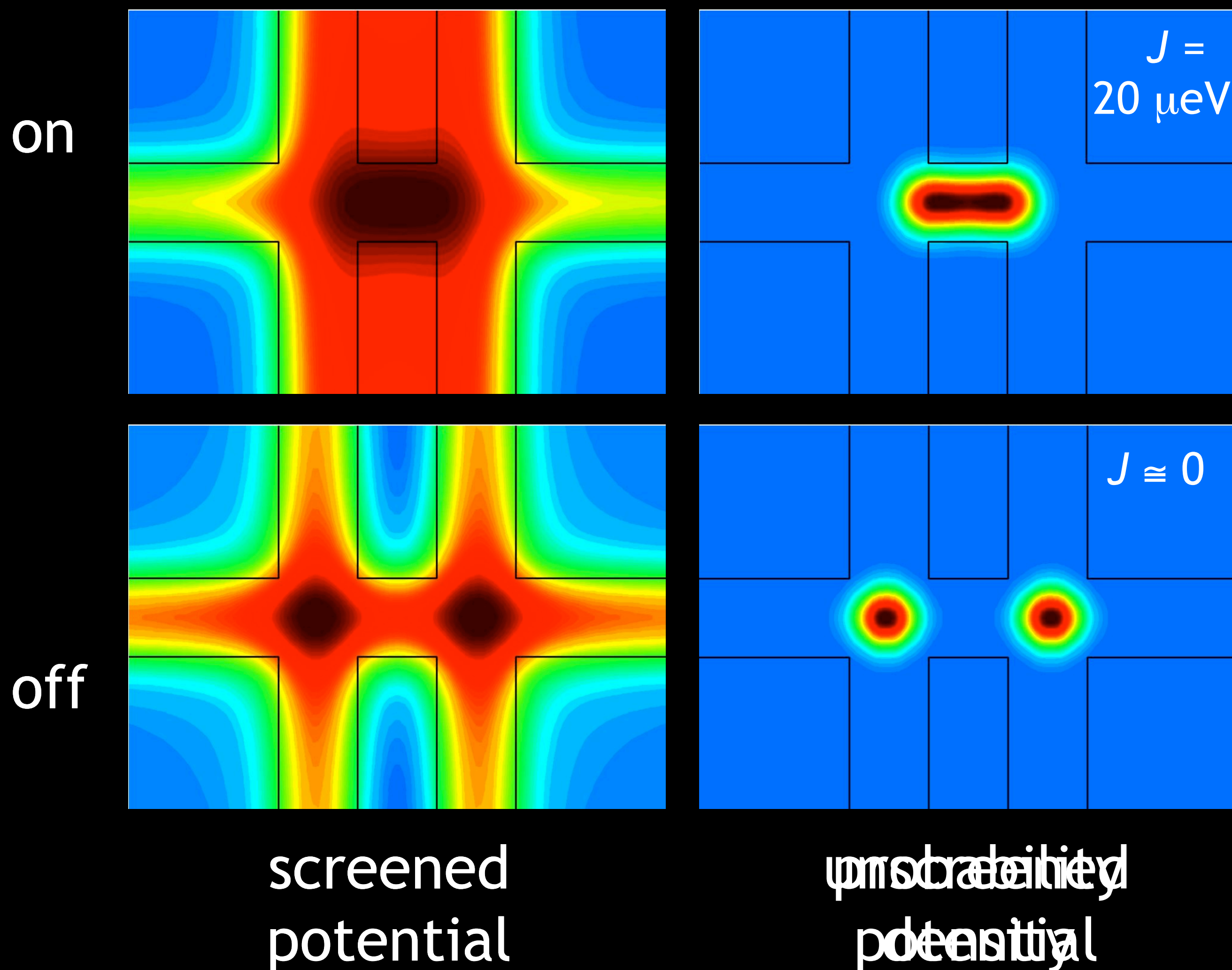
$$\text{SWAP:} \quad \text{Int}[J(t)\ dt] = \pi\hbar$$

=> CNOT

Simulation: Coupled Qubits in Silicon

(Friesen, Rugheimer, Savage, et al., '03)





Exchange and CNOT

$$H_{eff} = J(t)\, \mathbf{s}_1 \cdot \mathbf{s}_2 \qquad U(t)|\psi\rangle = e^{\frac{i}{\hbar}\mathbf{S}_1\cdot\mathbf{S}_2 \int J(t)dt}$$

$$\mathbf{S}^2 = \mathbf{S}_1{}^2 + \mathbf{S}_2{}^2 + 2\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$\int J(t)dt = JT/\hbar = \theta$$

$$U = e^{i\theta \mathbf{S}_1\cdot\mathbf{S}_2} = \exp\left[i\theta \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] = e^{i\theta/4} U_{SWAP}$$

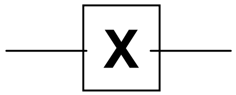
$$U_{CNOT} = e^{i(\pi/2)Z_1} e^{-i(\pi/2)Z_2} \sqrt{U_{SWAP}}\, e^{i(\pi/2)Z_1} \sqrt{U_{SWAP}}$$

One qubit operations

- **Single qubit rotations on the Bloch sphere**

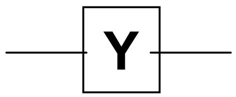
$$U|\psi\rangle = e^{-iHt/\hbar}|\psi\rangle$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

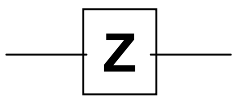


$$X|0\rangle = |1\rangle \qquad U = e^{-i\theta X/2}$$

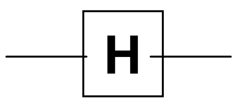
$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

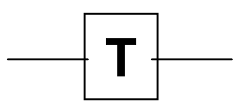


$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\pi/4) \end{pmatrix}$$

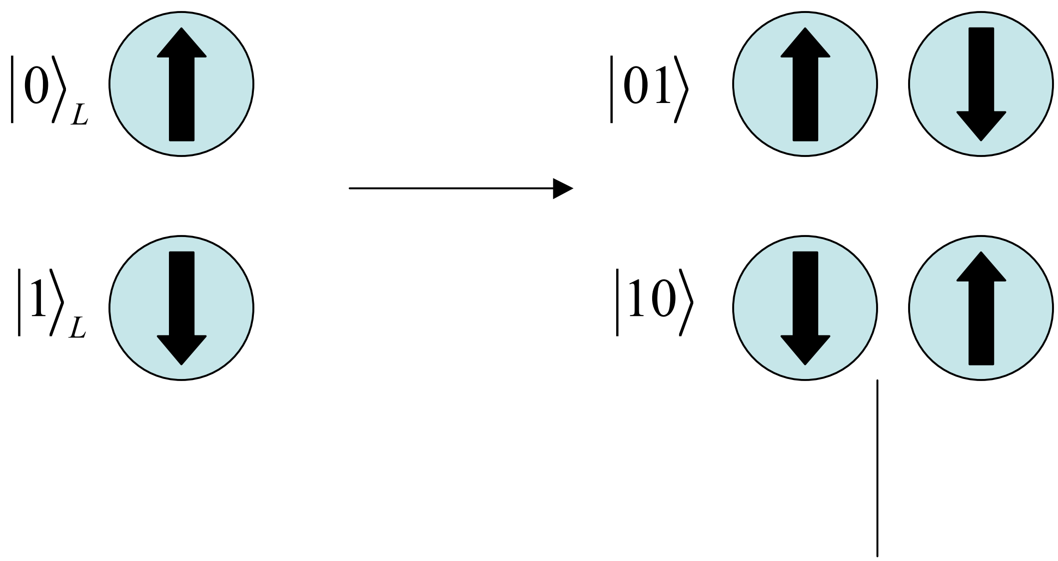


Physical realization?

- **ESR-AC Magnetic**
- **On-chip Magnetic**
- **Neighbor ferromagnetic dot**
- **encoded qubits**

Encoded qubits

Logical qubit **Physical qubits**



Doing a swap operation is the same as an X operation on the encoded space.

$$U_{SWAP}(1,2) = X|10\rangle = |01\rangle$$

Exchange is the same as rotation about X axis.

Initialization Schemes

- B-field
- Neighbor reference spin
- Optical pumping

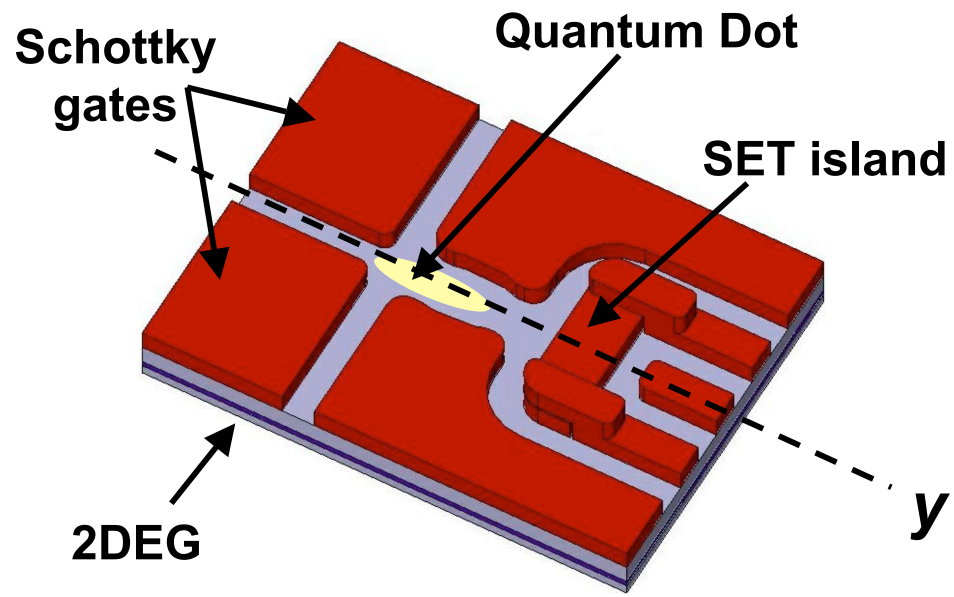
Readout Schemes

- Magnetic STM tip
- Spin-charge transduction
  - Spin-blockade transport measurement
  - Spin-orbital transduction

Device design for QD readout

[Friesen, Tahan, Joynt, Eriksson, *condmat*]

- Spin-dependent charge motion
- SET detection
- Microwave pumping

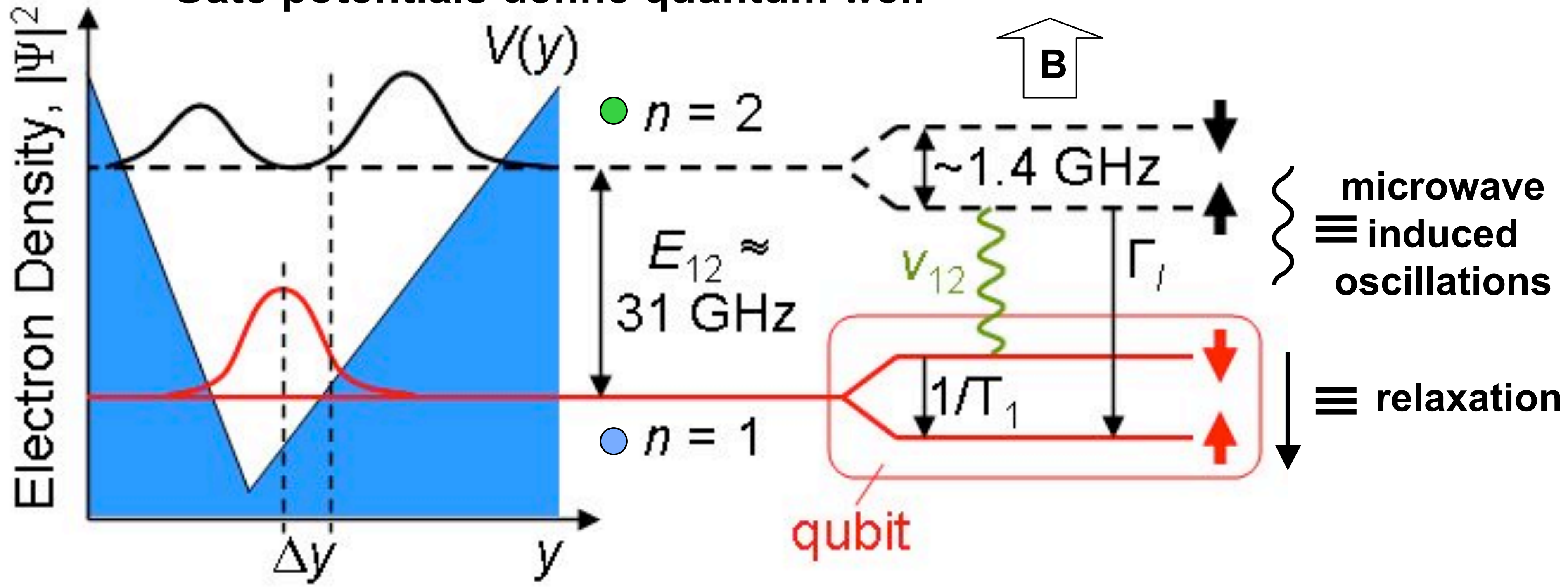


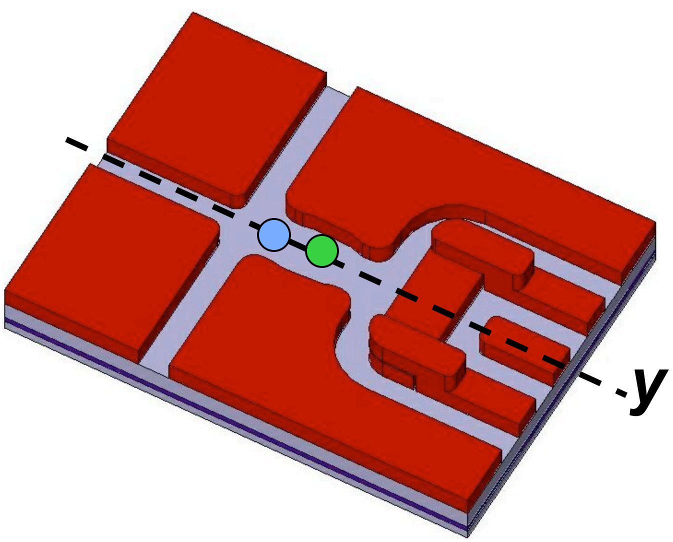
Fast readout and initialization is important for error correction

History…spin-charge transduction Loss/Divincenzo, Kane, …

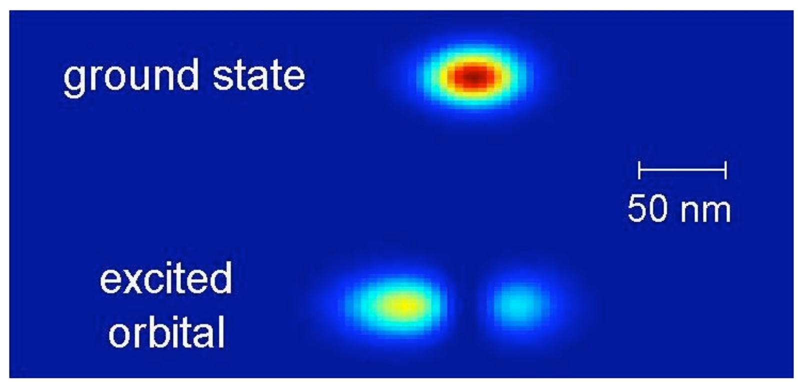
Charge movement in asymmetric well

Gate potentials define quantum well





• spin info to charge info via spin-dependent excitation



qc.physics.wisc.edu

QEC - Active

No cloning theorem: it is NOT possible to make a copy of an unknown quantum state

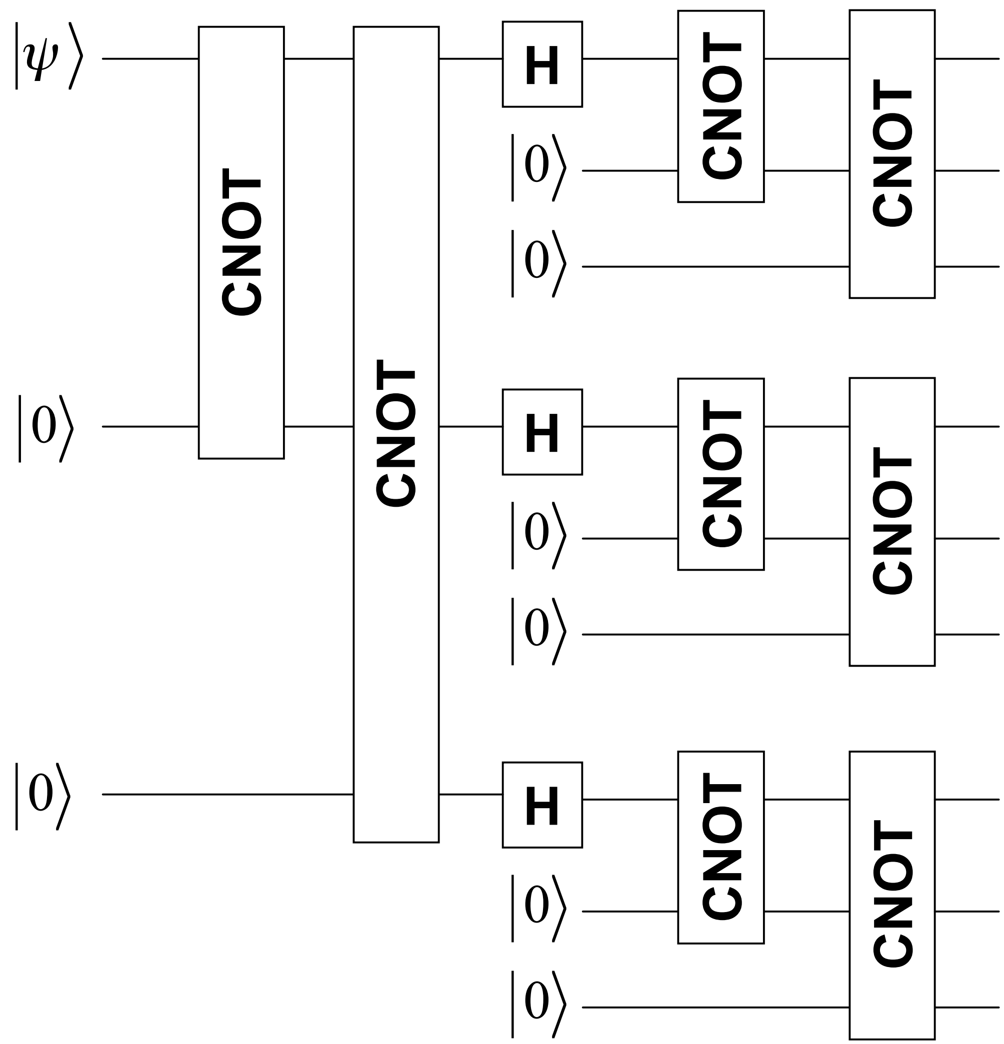
The Shor code: 9 qubits

phase flip code

$$|0\rangle \rightarrow |0\rangle_L = |+++\rangle$$
$$|1\rangle \rightarrow |1\rangle_L = |---\rangle$$

bit flip code

$$|0\rangle \rightarrow |0\rangle_L = |000\rangle$$
$$|1\rangle \rightarrow |1\rangle_L = |111\rangle$$



Threshold theorem

QEC - Passive

- Decoherence Free Subspaces
- Encoded qubits
- Uses a symmetry of the problem

Example: Collective dephasing

Error:

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow e^{i\theta}|1\rangle$$

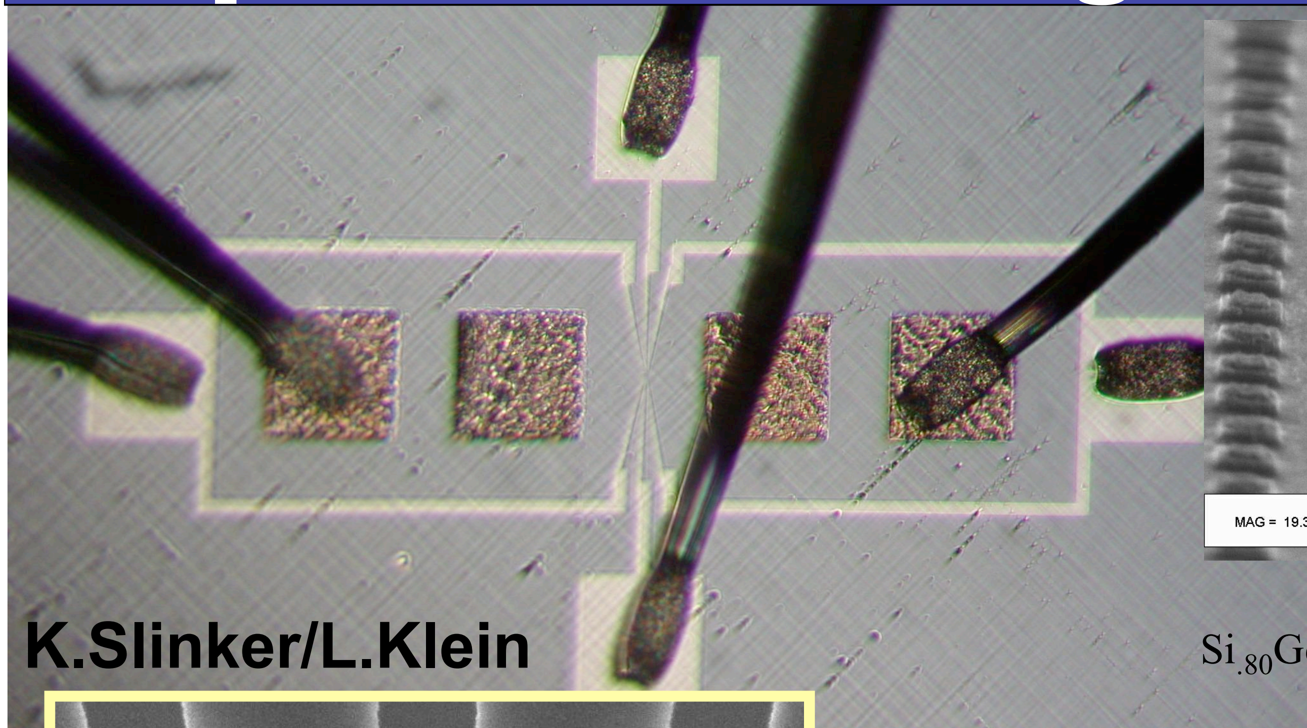
Encoding:

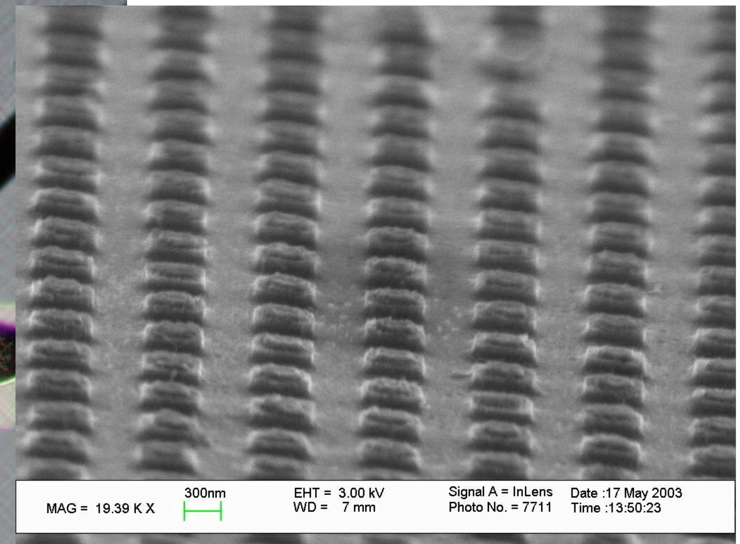
$$|0\rangle_L \rightarrow \frac{1}{\sqrt{2}}\left(|01\rangle - i|10\rangle\right)$$

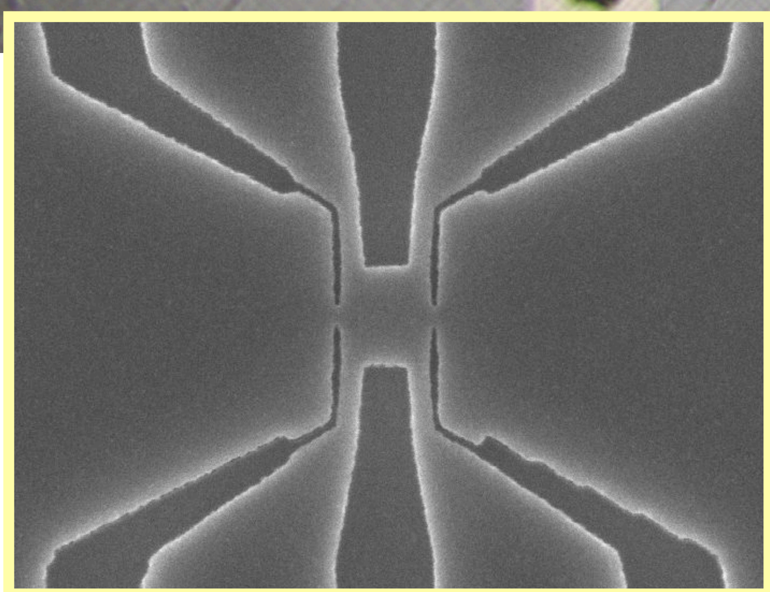
$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|01\rangle + i|10\rangle\right)$$

Relative phases do not change under collective dephasing

Experimental Progress







$Si_{.80}Ge_{.20}$

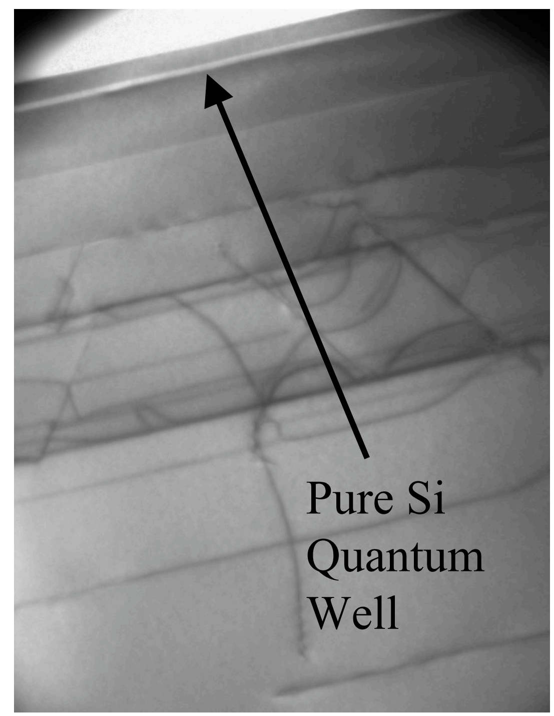
$Si_{.80}Ge_{.20}$

$Si_{.85}Ge_{.15}$

$Si_{.90}Ge_{.10}$

$Si_{.95}Ge_{.05}$

Si substrate



The end

- Quantum dot quantum computing in silicon
- http://qc.physics.wisc.edu/ for more information on this and all Wisconsin QC

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