

Spintronics and Spin-based quantum dot quantum computing



qc.physics.wisc.edu

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ECE746-Quantum Electronics, Guest Lecture
2:30pm, Oct. 26, 2004





Ahead of the curve

Spintronics

Datta/Das SPINFET paper

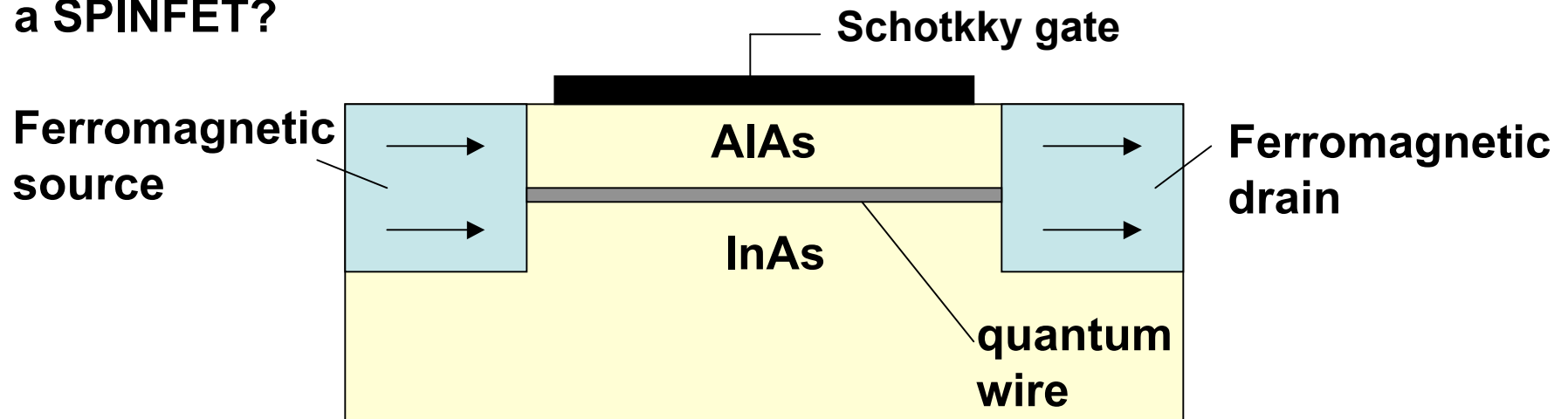
Quantum dot quantum computing

Loss/Divincenzo paper

QDQC in silicon at Wisconsin

Datta/Das: “Electric analog of the optoelectronic modulator”

- Ballistic electron transport through the quantum wire
- Spin polarized injection from ferromagnetic source
- Spin orbit coupling rotates the electron spin depending on the strength of the electric field and the length of the wire $\mathbf{B} = \mathbf{v} \times \mathbf{E}$
- At the drain, the electron’s transmission probability depends on the relative spin alignment
- Thus control of the source-to-drain current with the top gate
- a SPINFET?



UW-Madison Solid-State Quantum Computing

Mark Eriksson (Physics)

Robert Blick (ECE)

Sue Coppersmith (Physics)

Robert Joynt (Physics)

Max Lagally (Materials Science)

Dan van der Weide (ECE)

Mark Friesen (Materials Science and Physics)

Don Savage (Materials Science)

Levente Klein (Physics)

Shaolin Liao (Materials Science)

Keith Slinker (Physics)

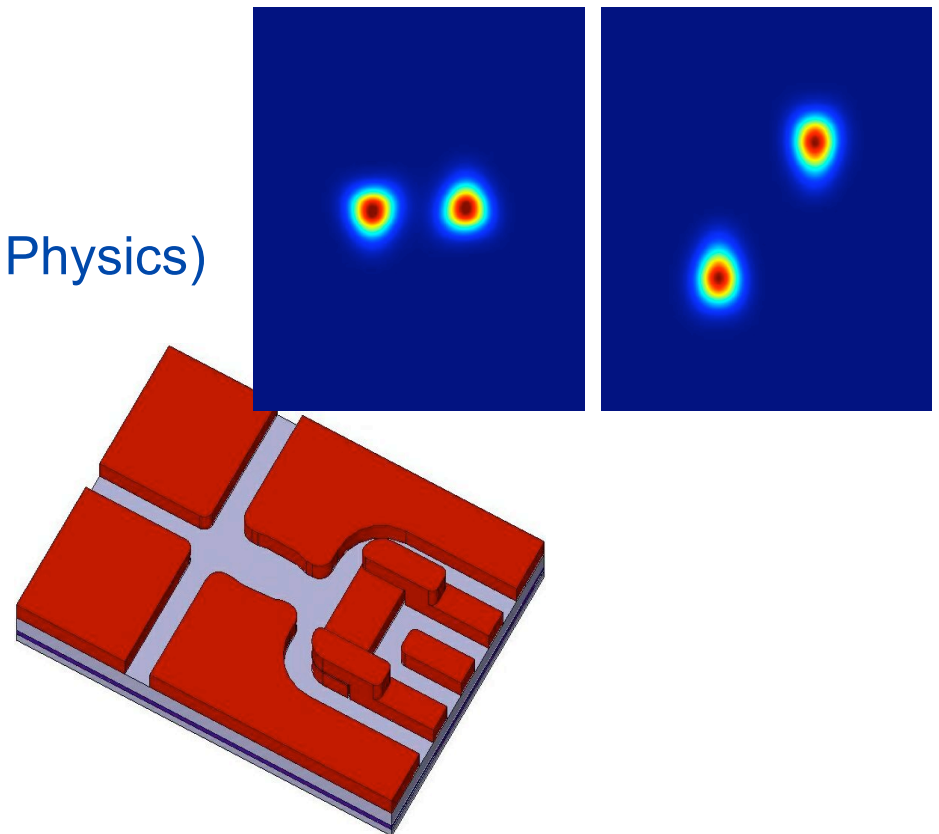
Charles Tahan (Physics)

Jim Truitt (ECE)

Kristin Morgenstern (Physics)

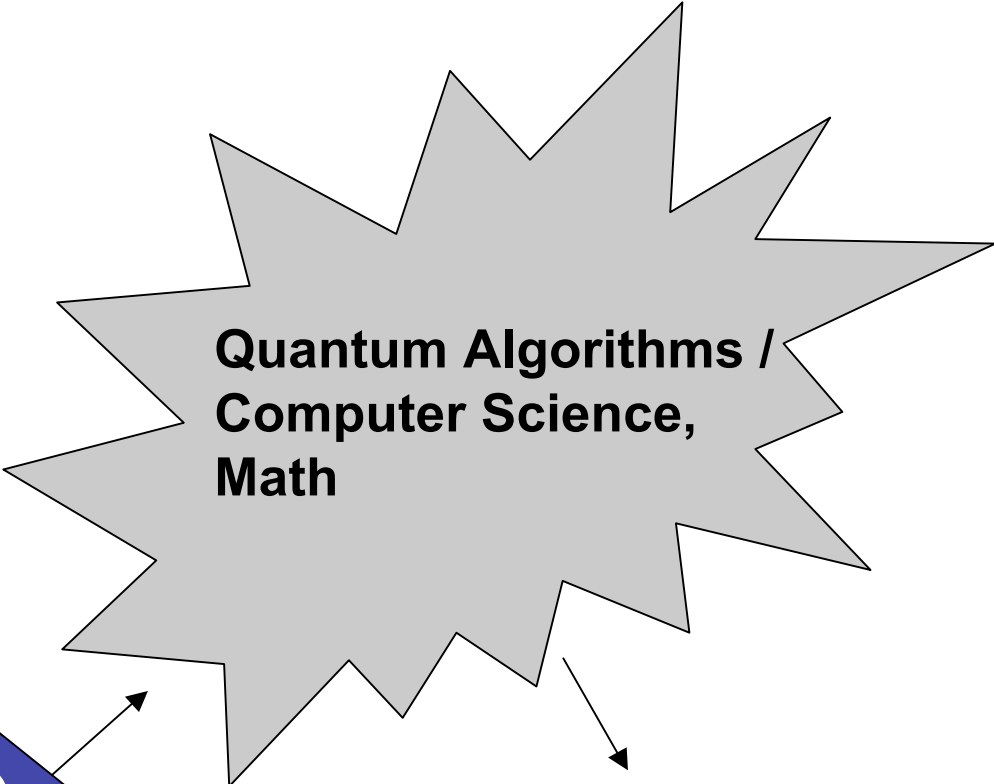
Srijit Goswami (Physics)

Diu Ngeihm (Physics)



Quantum Computing

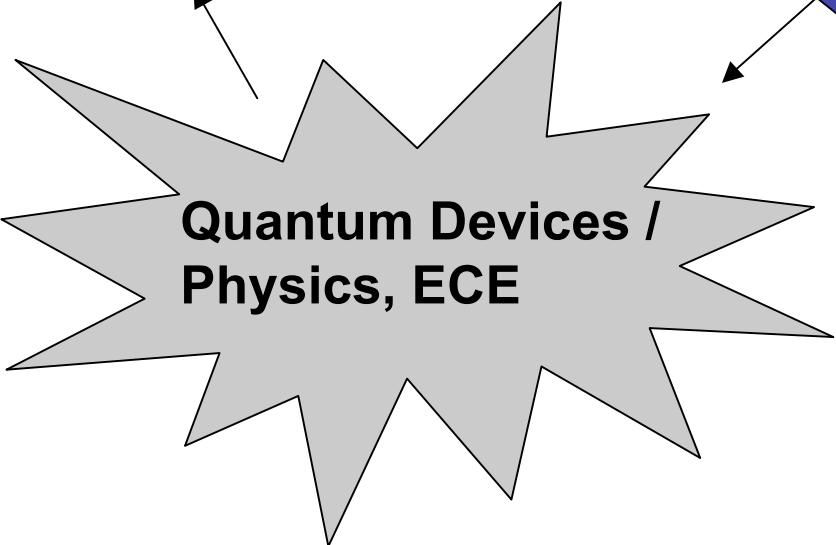
**Quantum Algorithms /
Computer Science,
Math**



**New paradigm for
information science**

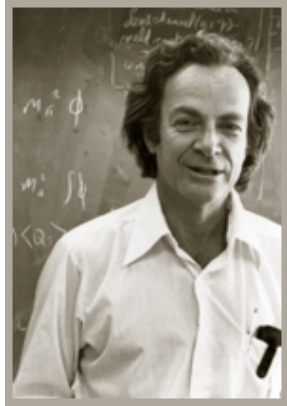
Quantum Technology

**Quantum Devices /
Physics, ECE**



Motivation

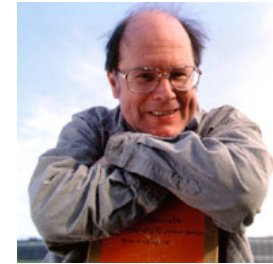
Shor Algorithm for prime factorization: killer application



R. Feynman



Peter Shor



Charles
Bennett



David
Deutsch

Error correction on a QC is possible

Quantum over classical

- Superposition - Parallelism
- Interference
- Entanglement

Building a QC

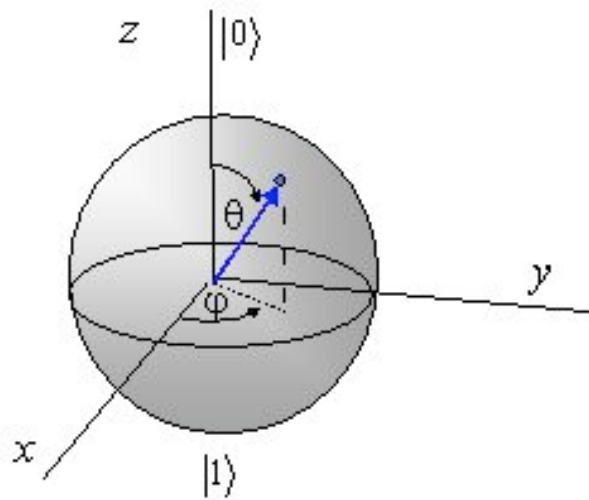
- Good, scalable qubit
- Two-qubit entanglement operation
- Fast readout/measurement of qubit
- Fast initialization / source of new qubits
- Quantum error correction
- Flying qubits

Formalism

Qubit: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
 “off” “on”

→ $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$
 “off AND on”

The Bloch sphere



$$w = w_0|0\rangle + w_1|1\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Quantum superposition

Multiple qubits:

$$|0\rangle \otimes |1\rangle \otimes |0\rangle =$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = [8 \times 1] \text{ dimensional Hilbert space}$$

Formalism

State vector formalism of quantum mechanics

$$H|\psi\rangle = E|\psi\rangle$$

Density matrix formalism

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] \quad \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\rho_{|0\rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

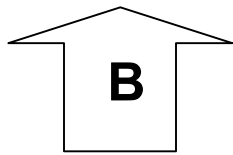
$$\rho_{|1\rangle} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_{|+\rangle} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\rho_{QC} = \rho_1 \otimes \rho_2 \otimes \rho_3 \otimes \dots$$

Good qubit: Spin

- Electronic or nuclear spin $\frac{1}{2}$
- Natural 2 level system
- Long coherence times
- Scalable (?) in semiconductor structures



$$\uparrow \rho_{|0\rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\downarrow \rho_{|1\rangle} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \rho_{|+\rangle} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Example:

$$T_1 \gg T_2 \quad t = 0$$

$$\rightarrow \rho = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$T_1 > t > T_2$$

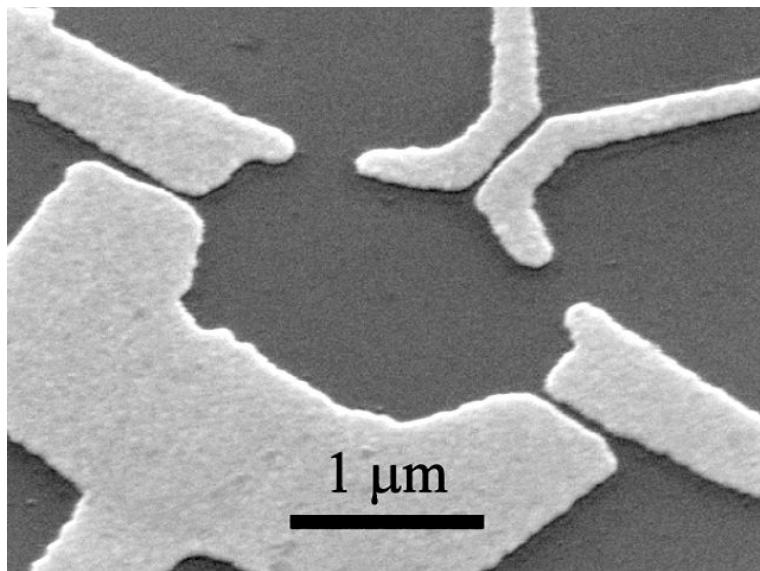
$$? \rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$t > T_1$$

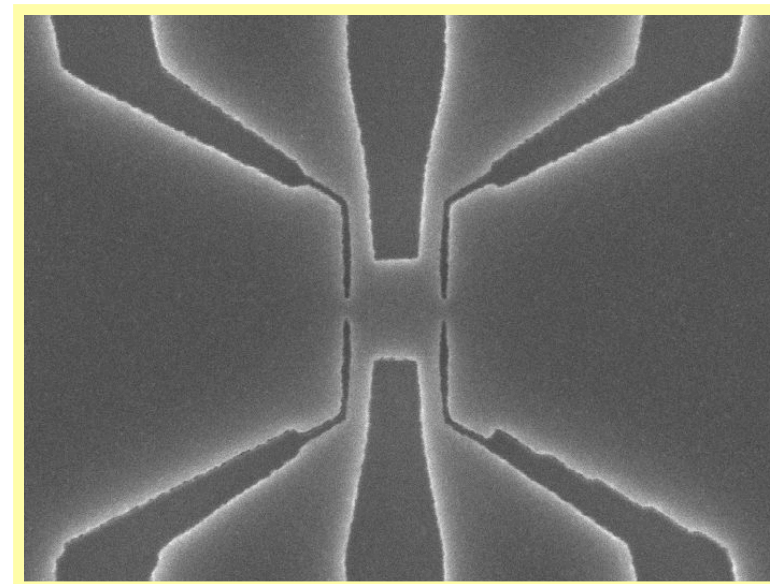
$$\uparrow \rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Quantum Dot Architectures

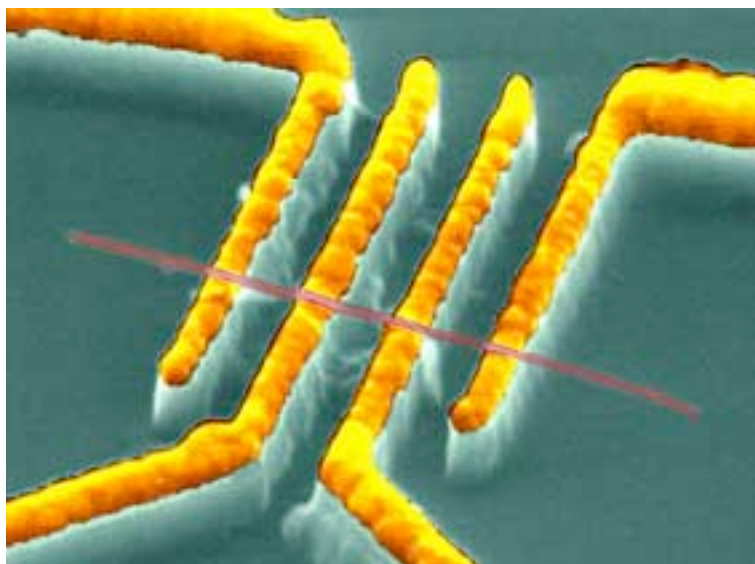
GaAs/
AlGaAs



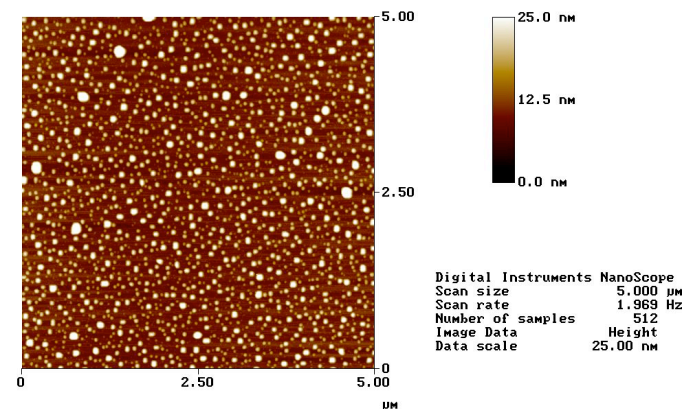
Si/SiGe



Carbon Nanotubes



Ge Huts



si25ge75.1a

Wisconsin QDQC design

1. Gated Quantum Dot QC (Loss & DiVincenzo)

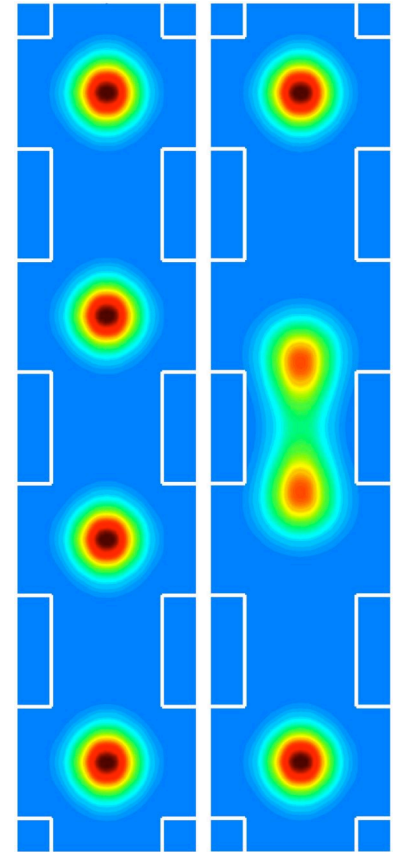
- 1 electron spin = 1 qubit
- Self aligning to gates (no need to align to donors)
- Fast operations through *Heisenberg exchange*
- Scalable (hopefully)

2. Silicon

- Long decoherence times ($T_2 \sim$ milliseconds for P: ^{28}Si)
- Low spin-orbit coupling
- Spin-zero nuclei ^{28}Si

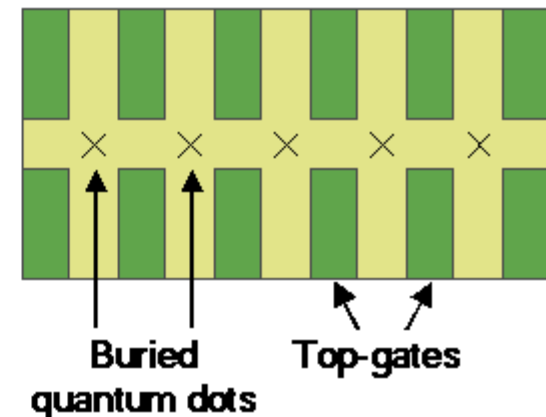
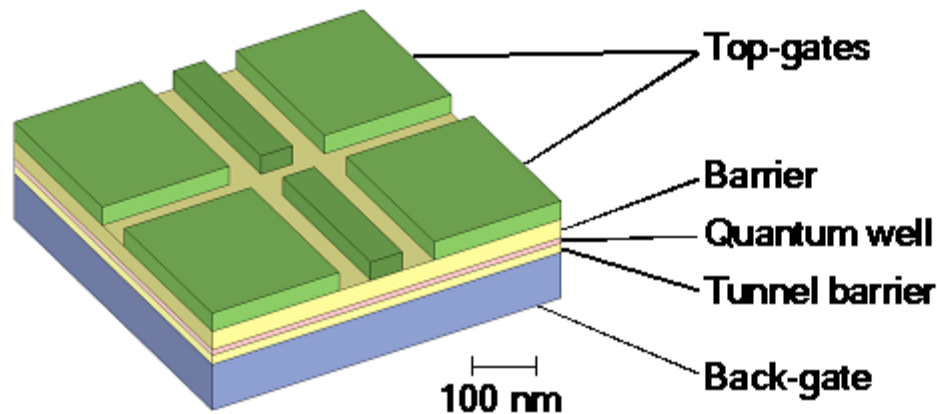
3. Back-gate

- Size-independent loading and well-screened manipulation of dots



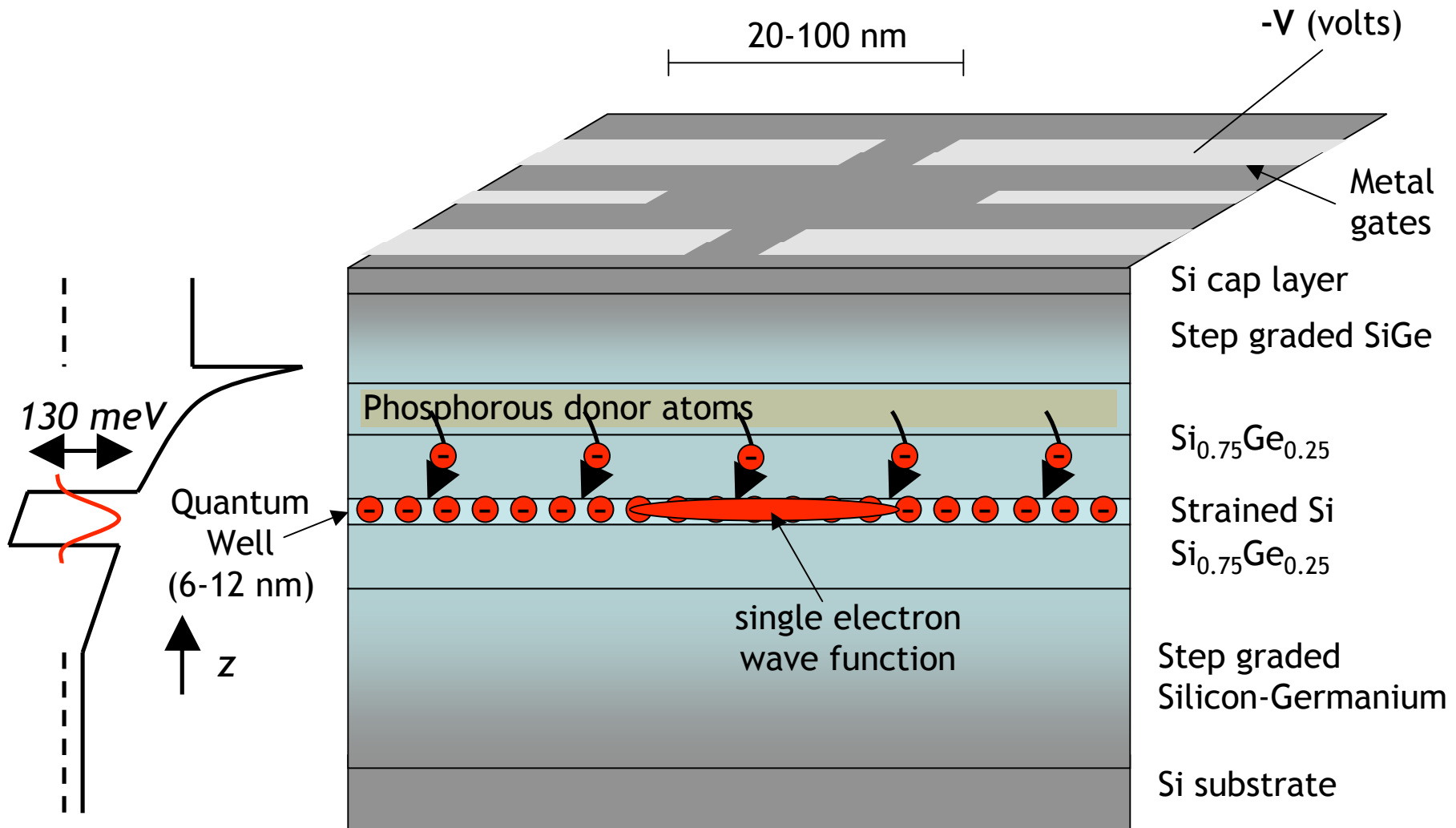
[Freisen, *et.al.*, APL]

[Freisen, *et.al.*, PRB]



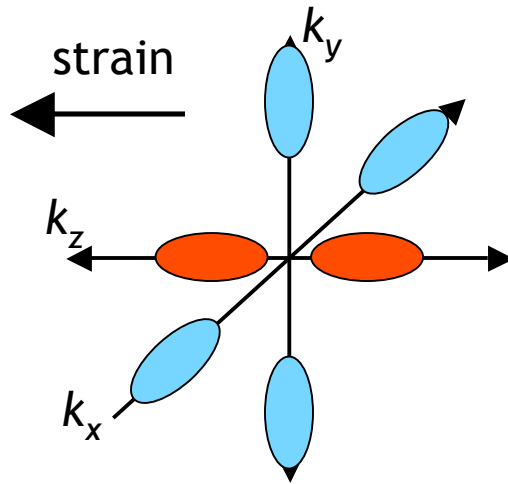
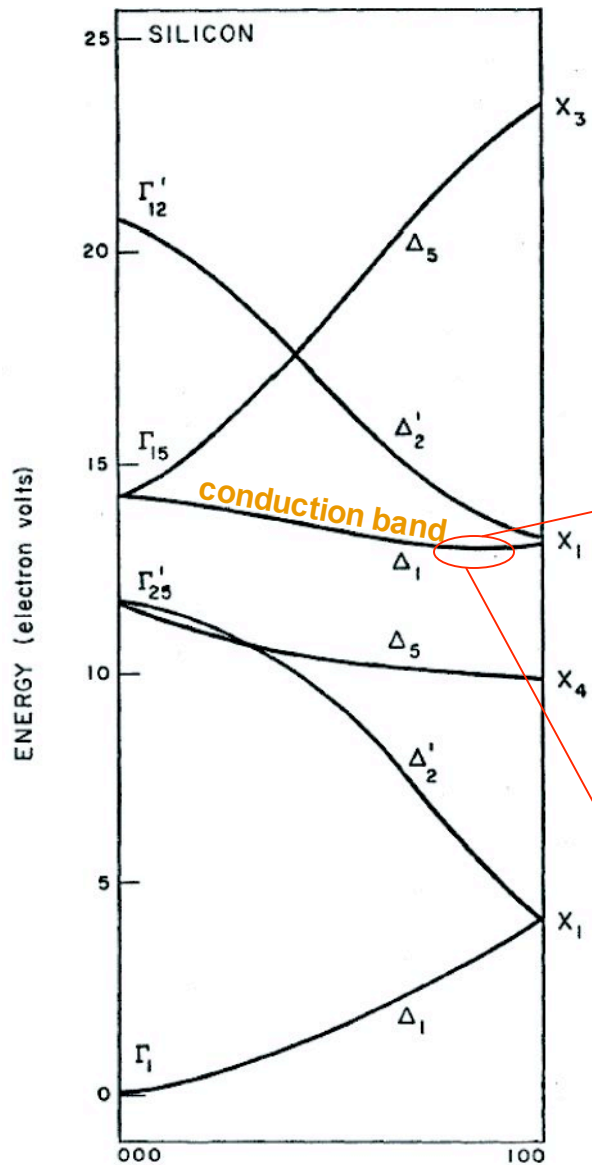
A quantum well quantum dot

Goal: a single electron tunably confined vertically and horizontally in a semiconductor nanostructure

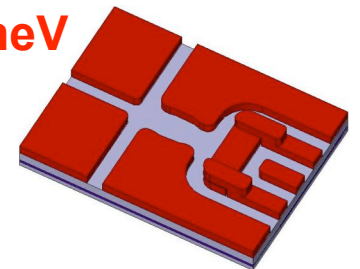
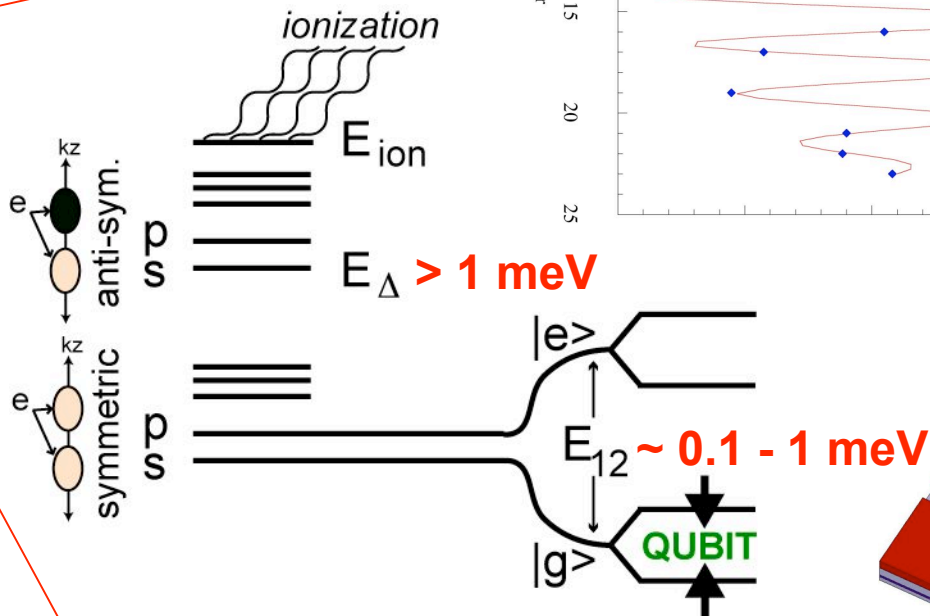
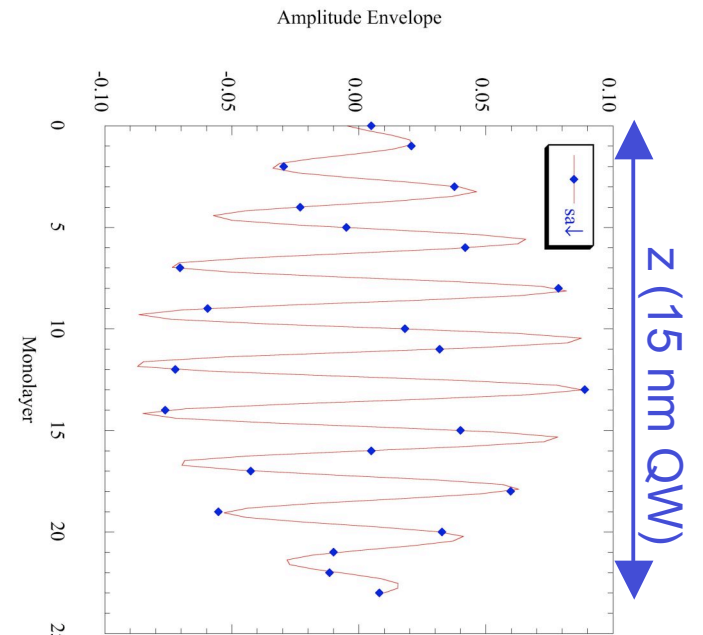


Details...

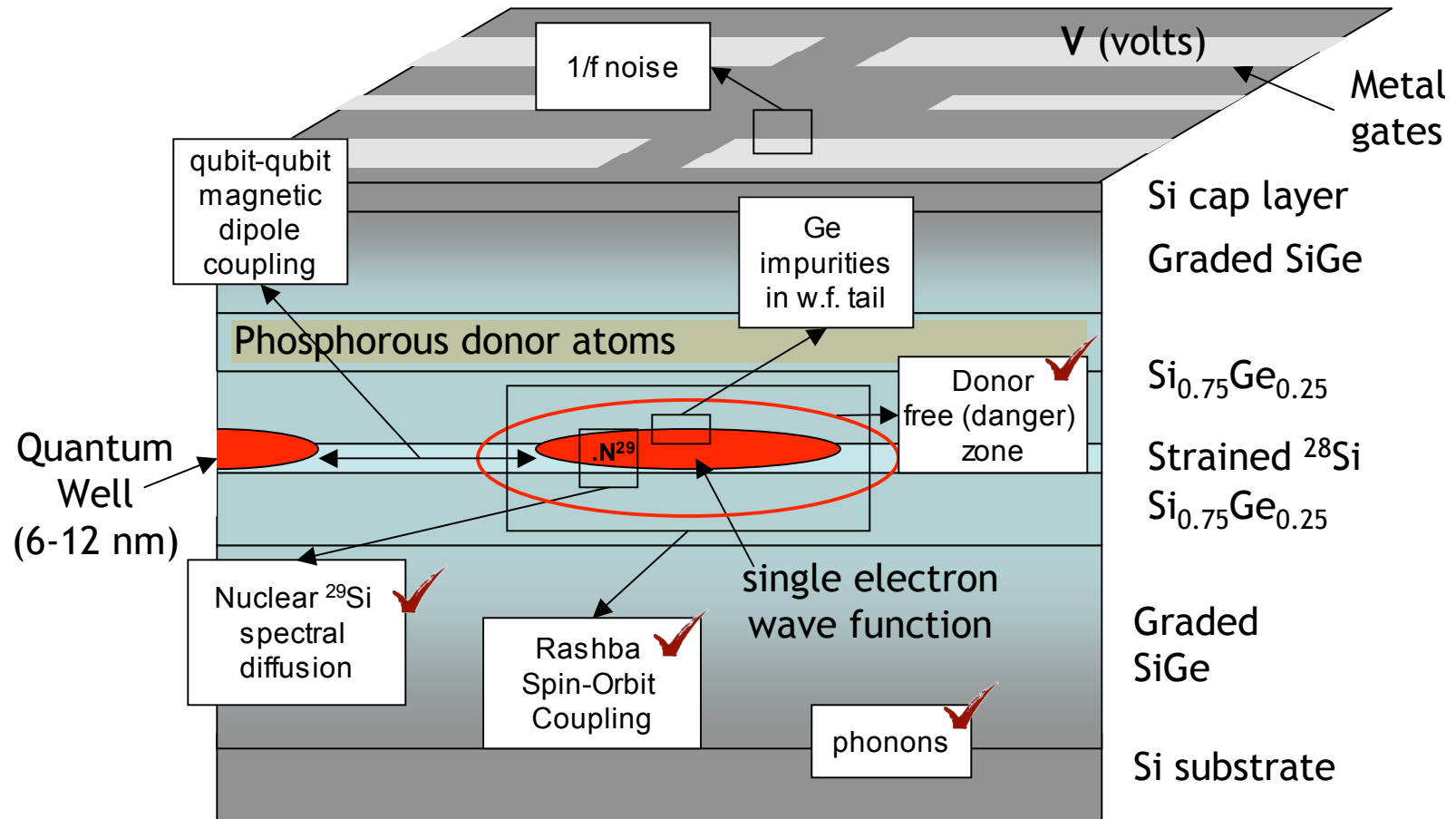
$$|\Psi\rangle = \text{Envelope} \times \text{Bloch}$$



Bloch functions



Decoherence

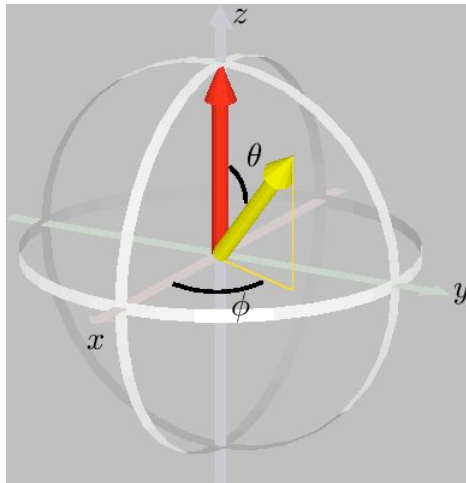


Universal QC

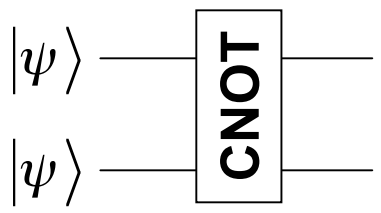
A *universal* set of gates can compute an arbitrary function (e.g. NAND for classical computation)

Quantum Algorithms/
Computer Science,
Math

Single qubit gates and **CNOT** are a universal set of gates for quantum computation.



$$\begin{aligned} \text{cnot}|00\rangle &= |00\rangle \\ \text{cnot}|01\rangle &= |01\rangle \\ \text{cnot}|10\rangle &= |11\rangle \\ \text{cnot}|11\rangle &= |10\rangle. \end{aligned}$$



A quantum circuit diagram showing two input qubits, both labeled $|\psi\rangle$, entering a rectangular box labeled "CNOT". Two output lines emerge from the box.

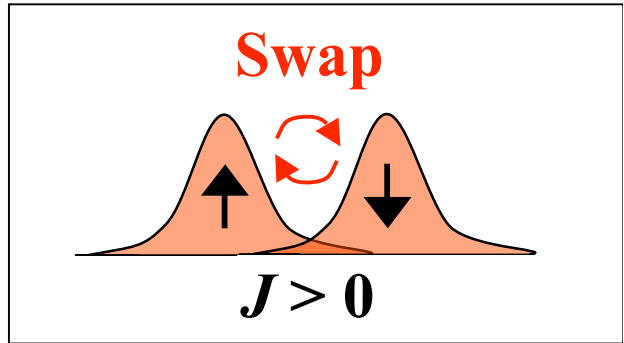
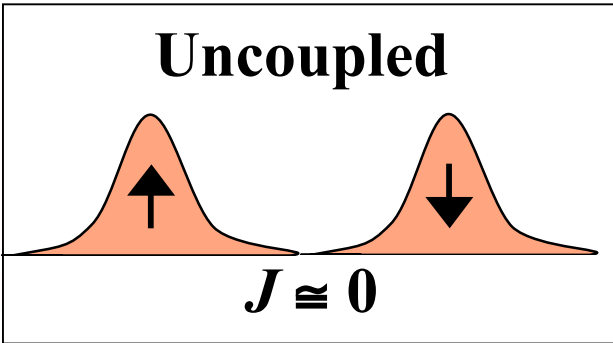
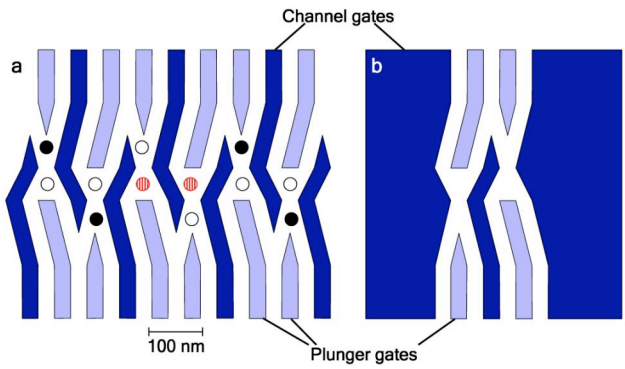
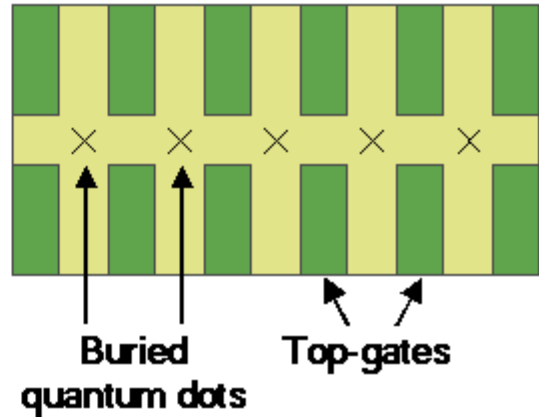
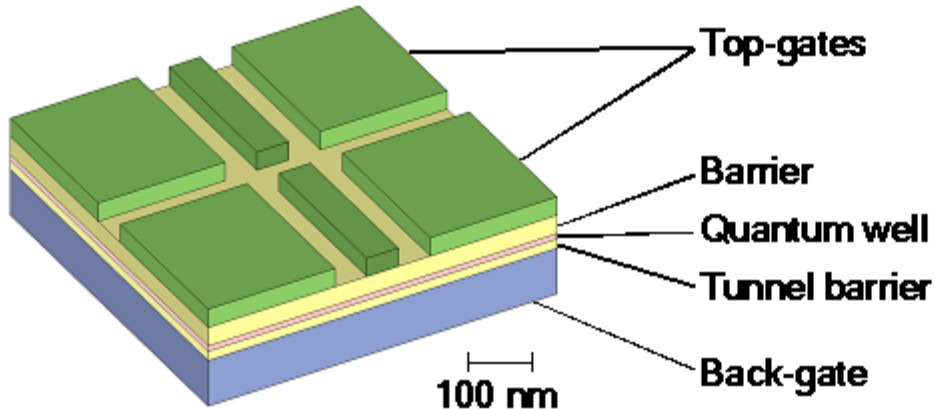
$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Entanglement

A quantum state of N qubits that cannot be written as a N-tensor product is said to be entangled.

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \neq |?\rangle_1 \otimes |?\rangle_2$$

Exchange and CNOT



H_2 quantum dots $\rightarrow H_{\text{eff}} = J \mathbf{s}_1 \cdot \mathbf{s}_2$

SWAP doesn't entangle but Sqrt[SWAP] does.

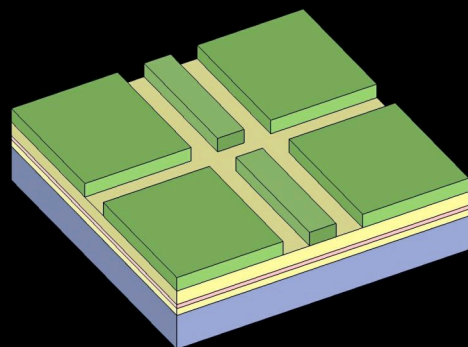
SWAP: $\text{Int}[J(t) dt] = \pi \hbar$

\Rightarrow CNOT

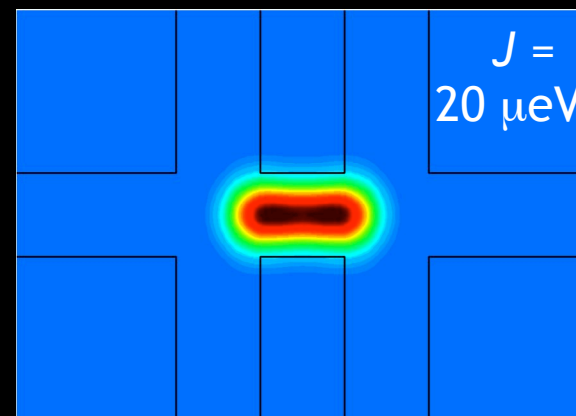
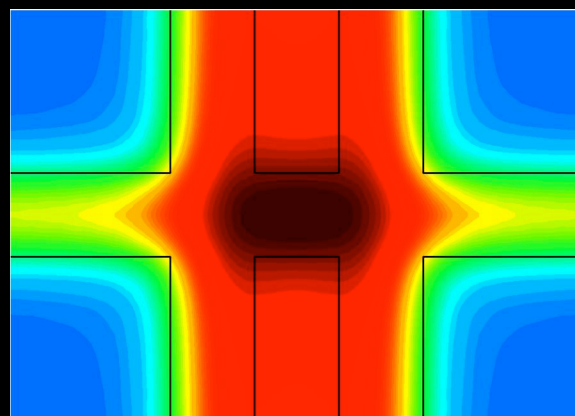
Simulation:

Coupled Qubits in Silicon

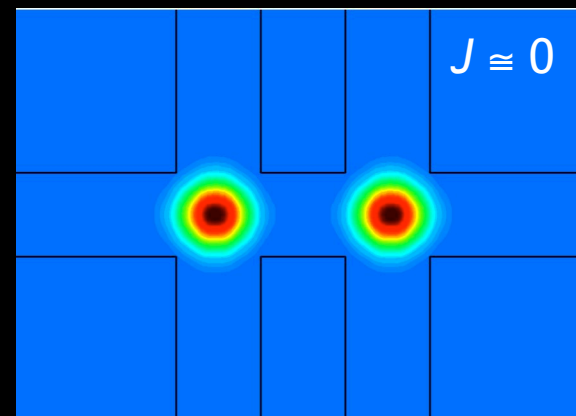
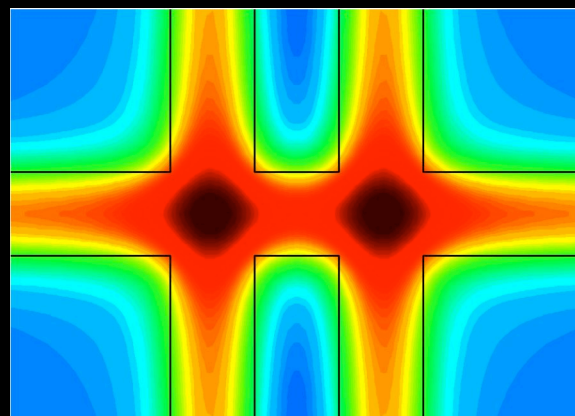
(Friesen, Rugheimer, Savage, et al., '03)



on



off



screened
potential

unscreened
potential

Exchange and CNOT

$$H_{\text{eff}} = J(t) \mathbf{s}_1 \cdot \mathbf{s}_2 \quad U(t)|\psi\rangle = e^{\frac{i}{\hbar} \mathbf{S}_1 \cdot \mathbf{S}_2 \int J(t) dt}$$

$$\mathbf{S}^2 = \mathbf{S}_1^2 + \mathbf{S}_2^2 + 2\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$\int J(t) dt = JT / \hbar = \theta$$

$$U = e^{i\theta \mathbf{S}_1 \cdot \mathbf{S}_2} = \exp \left[i\theta \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] = e^{i\theta/4} U_{\text{SWAP}}$$

$$U_{\text{CNOT}} = e^{i(\pi/2)Z_1} e^{-i(\pi/2)Z_2} \sqrt{U_{\text{SWAP}}} e^{i(\pi/2)Z_1} \sqrt{U_{\text{SWAP}}}$$

One qubit operations

- Single qubit rotations on the Bloch sphere

$$U|\psi\rangle = e^{-iHt/\hbar}|\psi\rangle$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{--- [X] ---}$$

$$X|0\rangle = |1\rangle$$

$$U = e^{-i\theta X/2}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{--- [Y] ---}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{--- [Z] ---}$$

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{--- [H] ---}$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

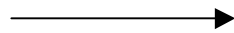
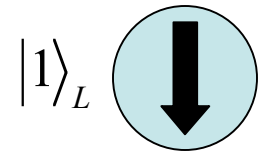
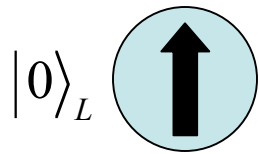
$$T = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\pi/4) \end{pmatrix} \quad \text{--- [T] ---}$$

Physical realization?

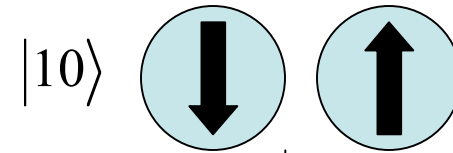
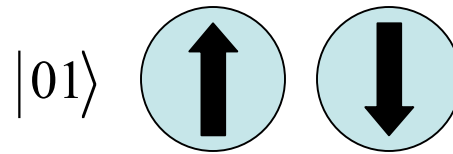
- ESR-AC Magnetic
- On-chip Magnetic
- Neighbor ferromagnetic dot
- encoded qubits

Encoded qubits

Logical qubit



Physical qubits



Doing a swap operation is the same as an X operation on the encoded space.

$$U_{SWAP}(1,2) = X|10\rangle = |01\rangle$$

Exchange is the same as rotation about X axis.

Initialization Schemes

- B-field
- Neighbor reference spin
- Optical pumping

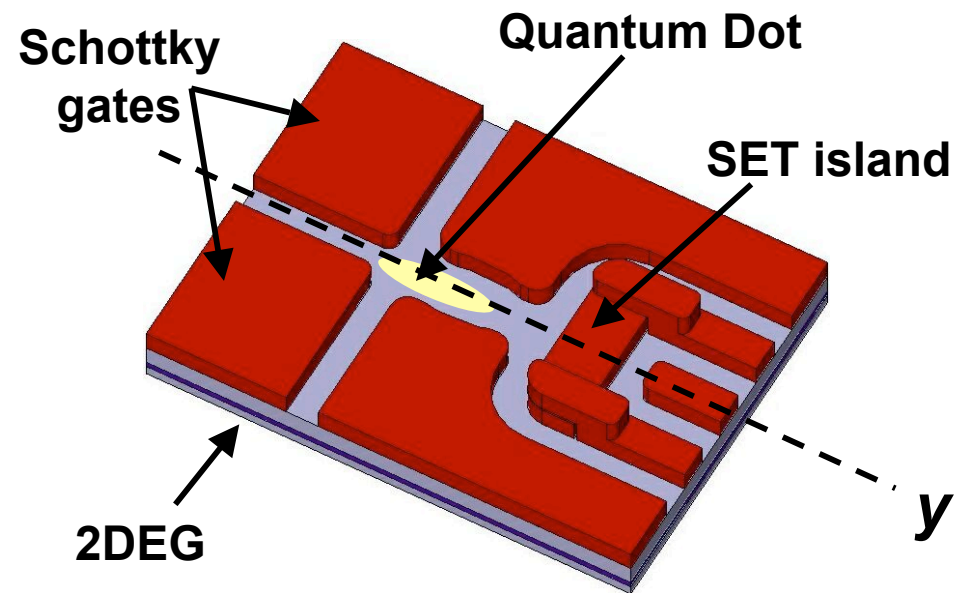
Readout Schemes

- Magnetic STM tip
- Spin-charge transduction
 - Spin-blockade transport measurement
 - Spin-orbital transduction

Device design for QD readout

[Friesen, Tahan, Joynt, Eriksson, *condmat*]

- Spin-dependent charge motion
- SET detection
- Microwave pumping

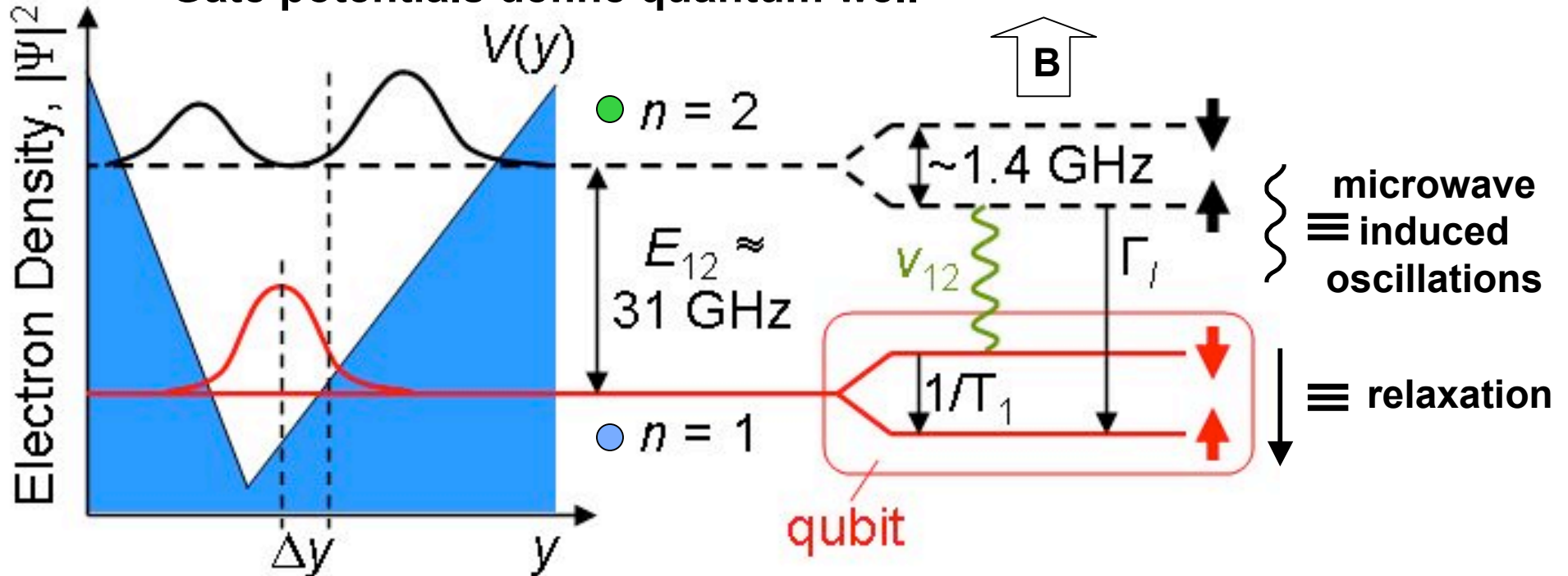


Fast readout and initialization is important for error correction

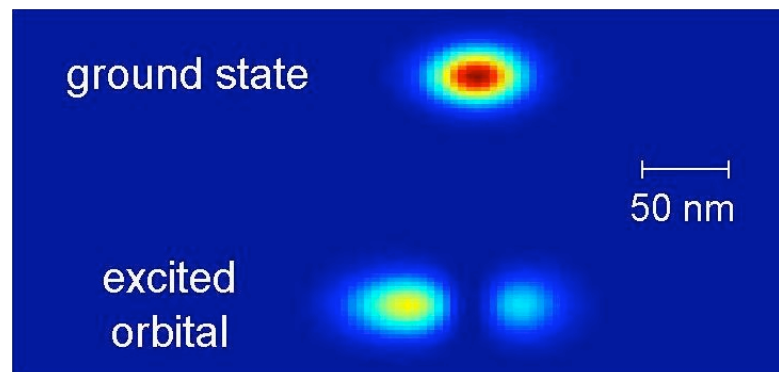
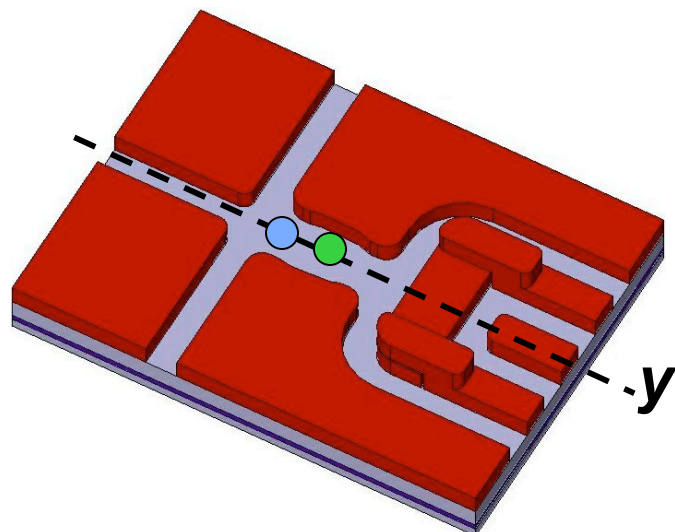
History... spin-charge transduction
Loss/Divincenzo,
Kane, ...

Charge movement in asymmetric well

Gate potentials define quantum well



- spin info to charge info via spin-dependent excitation



QEC - Active

No cloning theorem: it is NOT possible to make a copy of an unknown quantum state

The Shor code: 9 qubits

phase flip code

$$|0\rangle \rightarrow |0\rangle_L = |+++ \rangle$$

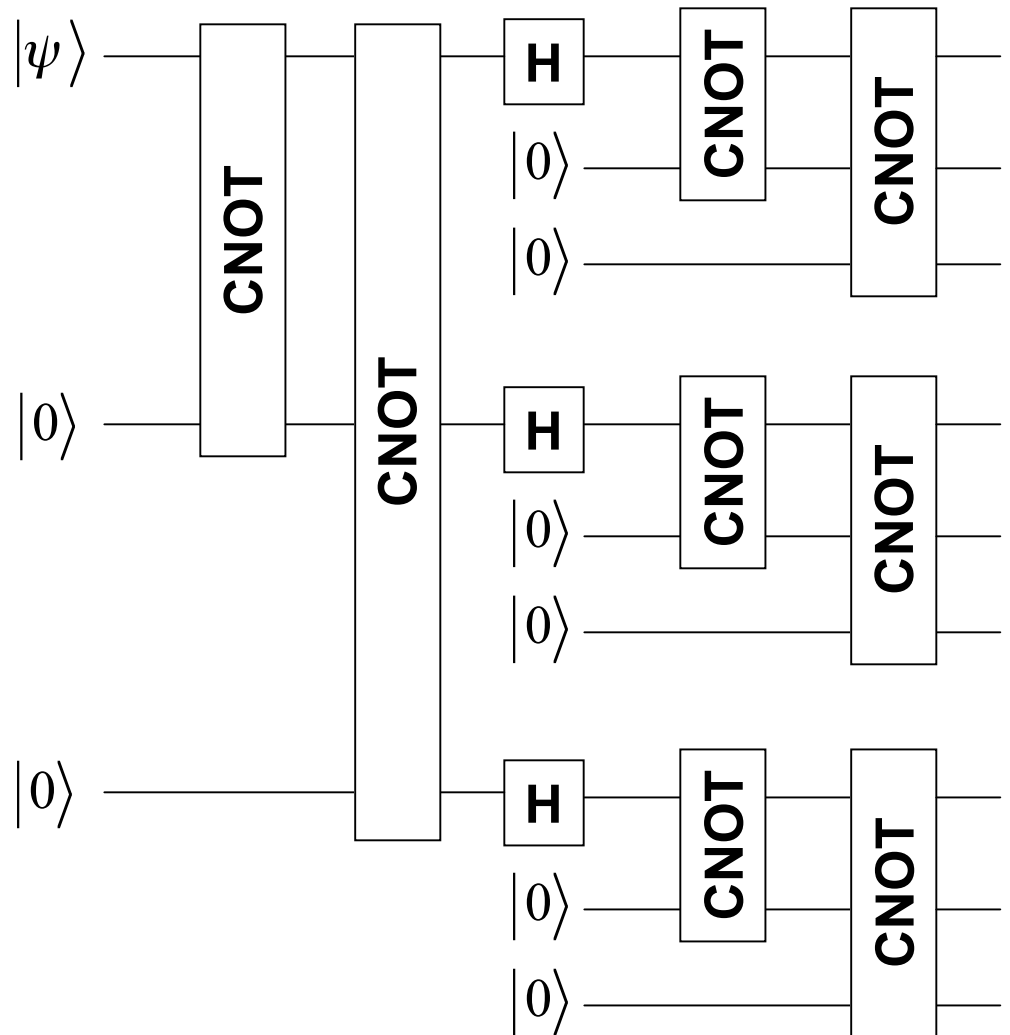
$$|1\rangle \rightarrow |1\rangle_L = |-- -- \rangle$$

bit flip code

$$|0\rangle \rightarrow |0\rangle_L = |000 \rangle$$

$$|1\rangle \rightarrow |1\rangle_L = |111 \rangle$$

Threshold theorem



QEC - Passive

- Decoherence Free Subspaces
- Encoded qubits
- Uses a symmetry of the problem

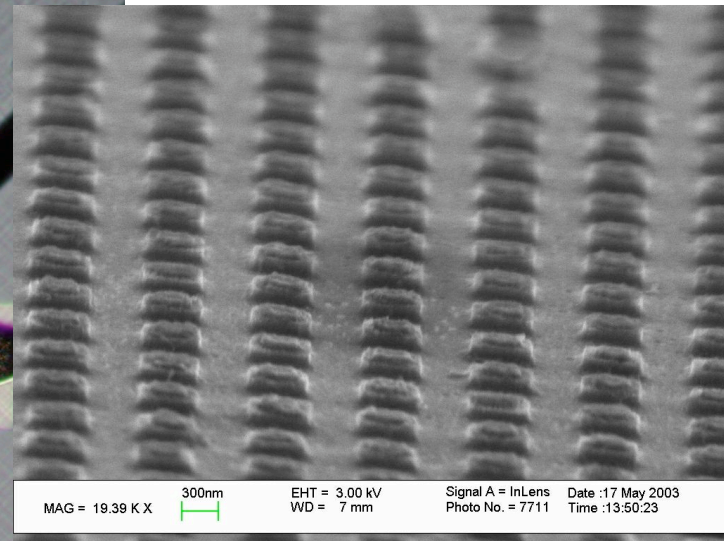
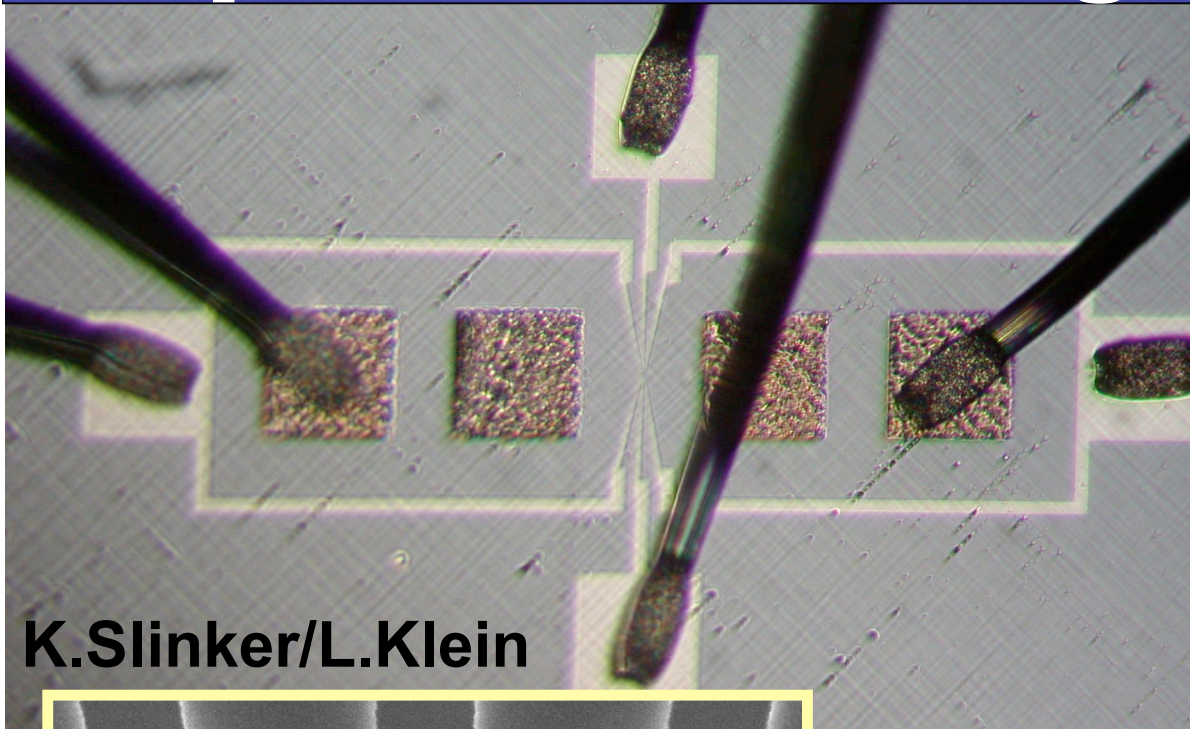
Example: Collective dephasing

Error: $|0\rangle \rightarrow |0\rangle$
 $|1\rangle \rightarrow e^{i\theta}|1\rangle$

Encoding: $|0\rangle_L \rightarrow \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle)$
 $|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle + i|10\rangle)$

**Relative phases
do not change
under collective
dephasing**

Experimental Progress



K.Slinker/L.Klein

$\text{Si}_{.80}\text{Ge}_{.20}$

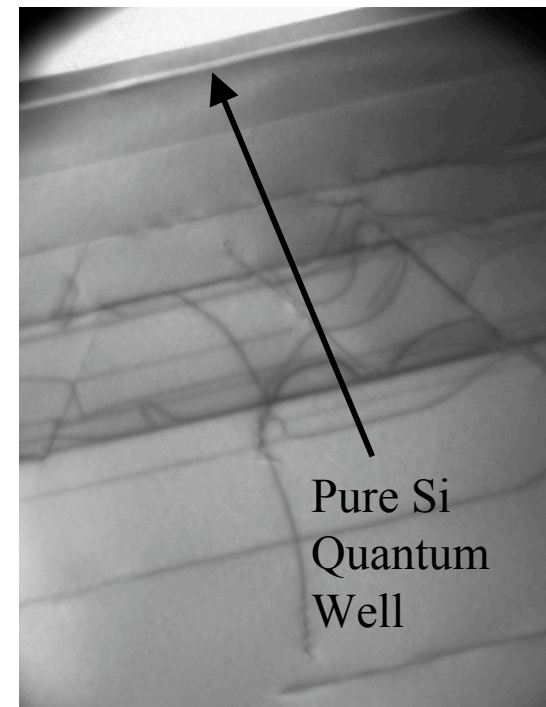
$\text{Si}_{.80}\text{Ge}_{.20}$

$\text{Si}_{.85}\text{Ge}_{.15}$

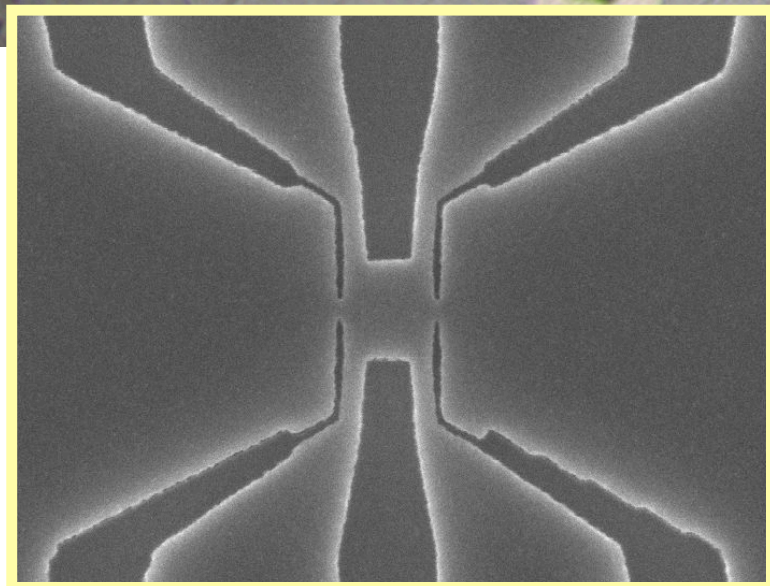
$\text{Si}_{.90}\text{Ge}_{.10}$

$\text{Si}_{.95}\text{Ge}_{.05}$

Si substrate



Pure Si
Quantum
Well



The end

- Quantum dot quantum computing in silicon
- <http://qc.physics.wisc.edu/> for more information on this and all Wisconsin QC

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