Quantum Dot Quantum Computers and other Entanglement-based devices

qc.physics.wisc.edu

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University of Wisconsin-Madison

Physical Chemistry Student Seminar
Nov. 7, 2004

Abstract

Information is physical. This rediscovery has lead to a paradigm shift in the fields of information theory and computer science. Perhaps the laws of nature, rather than the abstract world of 0s, 1s, and Turing machines, provide a more fundamental foundation for computing? This thesis lead to the discovery of quantum algorithms and quantum communication schemes which, in some cases, are more powerful than their classical counterparts. The key resource is quantum entanglement, the non-local interactions pervasive in quantum theory. Meanwhile, physicists are reaching the nanoscale limit, where dynamical control of quantum systems has, for the first time, become a possibility.

My perspective on this is that of an electron spin trapped in silicon. Despite the complexities, this quantum bit may be a great place to store and manipulate quantum information. I'm going to try and explain entanglement via a few physical situations for spin-1/2 particles, nature's natural qubits. Quantum states are fragile and must remain coherent long enough to do a computation. Here, quantum error correction schemes come into play. Of course, I'll also talk about the quantum computing architecture envisioned and currently being pursued by some of us in Madison.

The Hierarchy

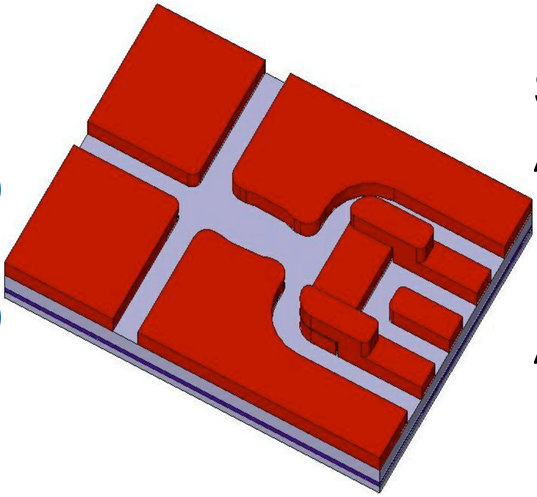
God?

Bob Joynt

Charlie

UW-Madison Solid-State Quantum Computing

Mark Eriksson (Physics)
Robert Blick (ECE)
Sue Coppersmith (Physics)
Robert Joynt (Physics)
Max Lagally (Materials Science)
Dan van der Weide (ECE)
Mark Friesen (Materials Science & Physics)
Don Savage (Materials Science)
Levente Klein (Physics)
Keith Slinker (Physics)
Charles Tahan (Physics)
Jim Truitt (ECE)
Srijit Goswami (Physics)
Kristin Lewis (Physics)
Cyrus Haselby (Physics)



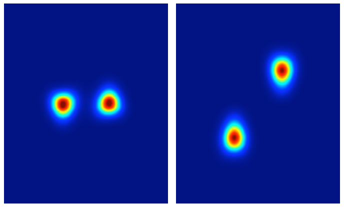
<u>Collaborators</u>

IBM:
Pat Mooney
Jack Chu

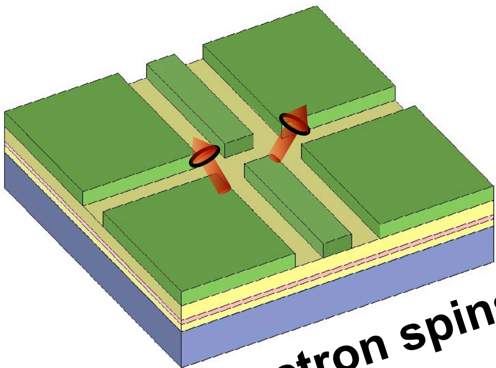
NEMO:
Gerhard Klimeck (Purdue)
Timothy Boykin (UAH)
Paul von Allmen (JPL)

Princeton University:
Steve Lyon
Alexei Tyryshkin

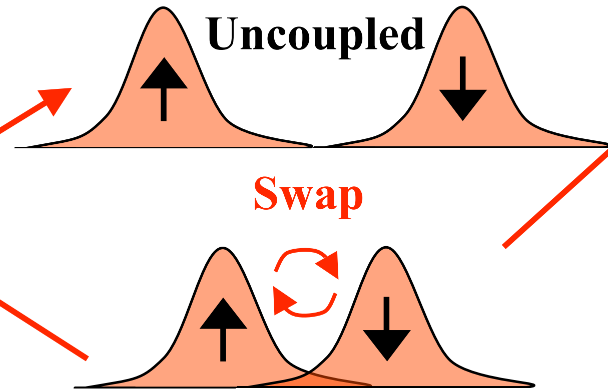
Dartmouth University:
Alex Rimberg



On Today's Schedule

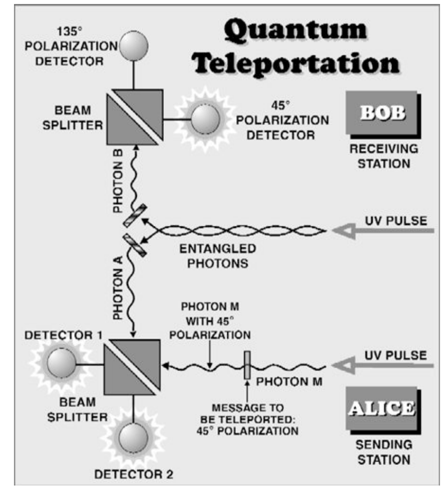


Electron spins

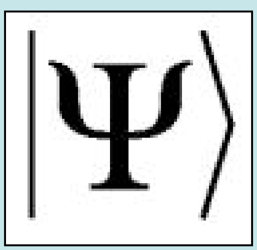


Exchange and Entanglement

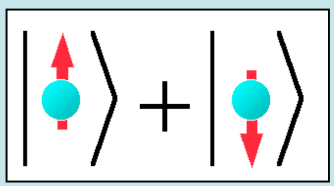
Quantum Information Devices



Decoherence and Quantum Control



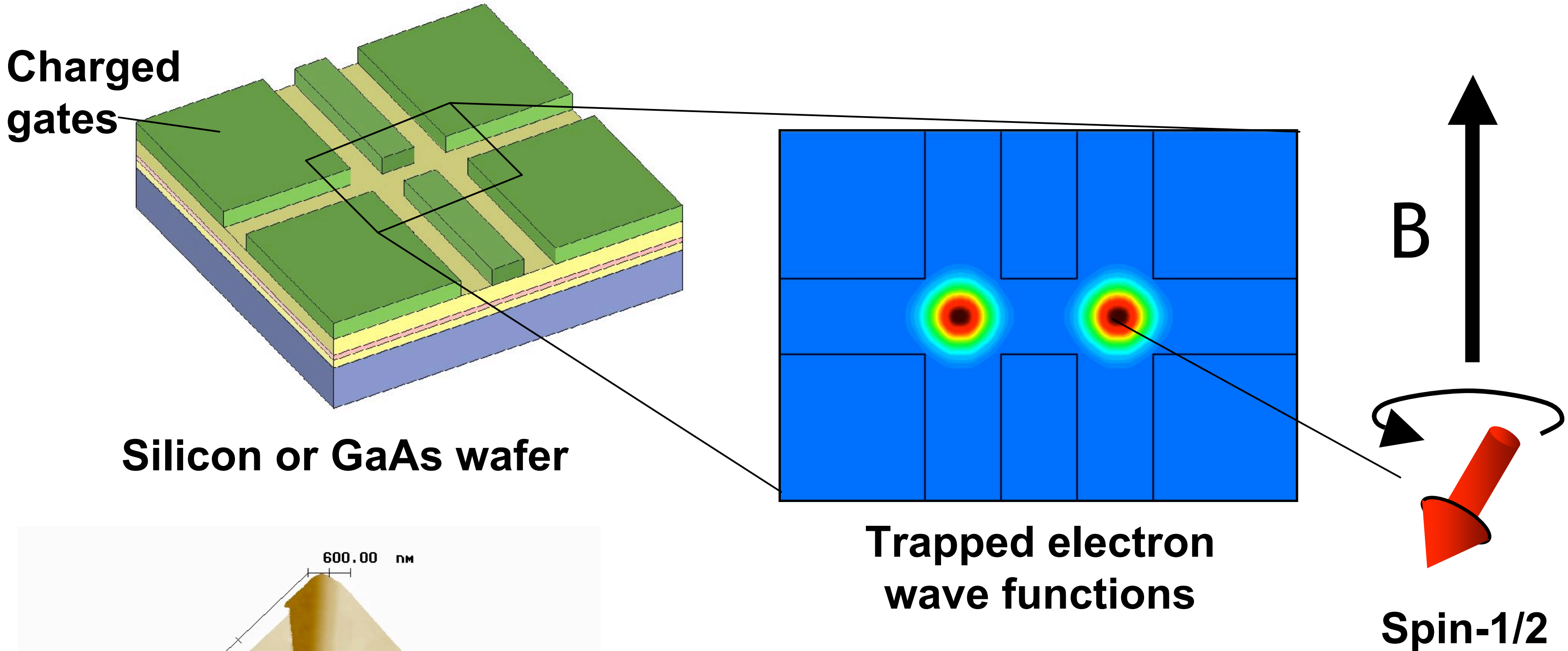
Quantum Information Theory vs. Quantum Technology

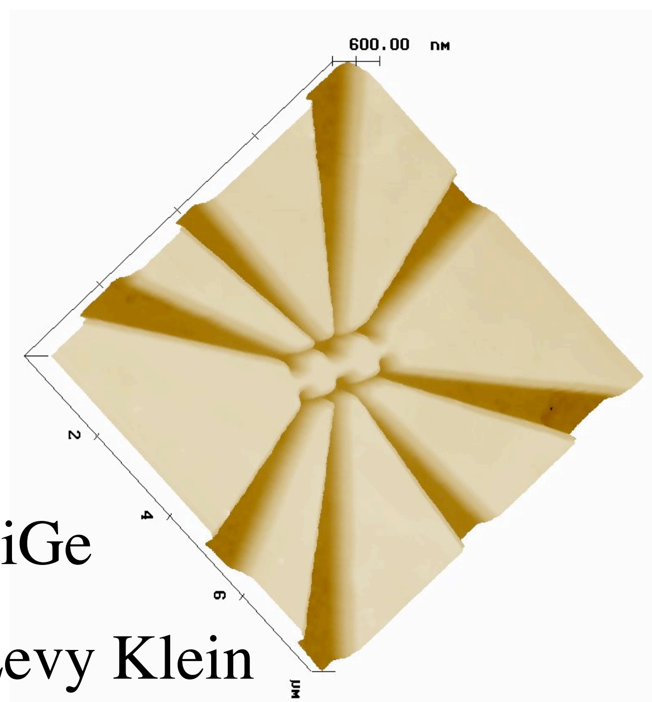


Quantum Computation

Entanglement

Spins in Quantum Dots



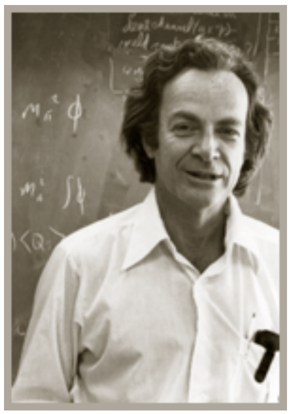


SiGe

Levy Klein

What are the lifetimes of excited spin states in semiconductors?

Motivation



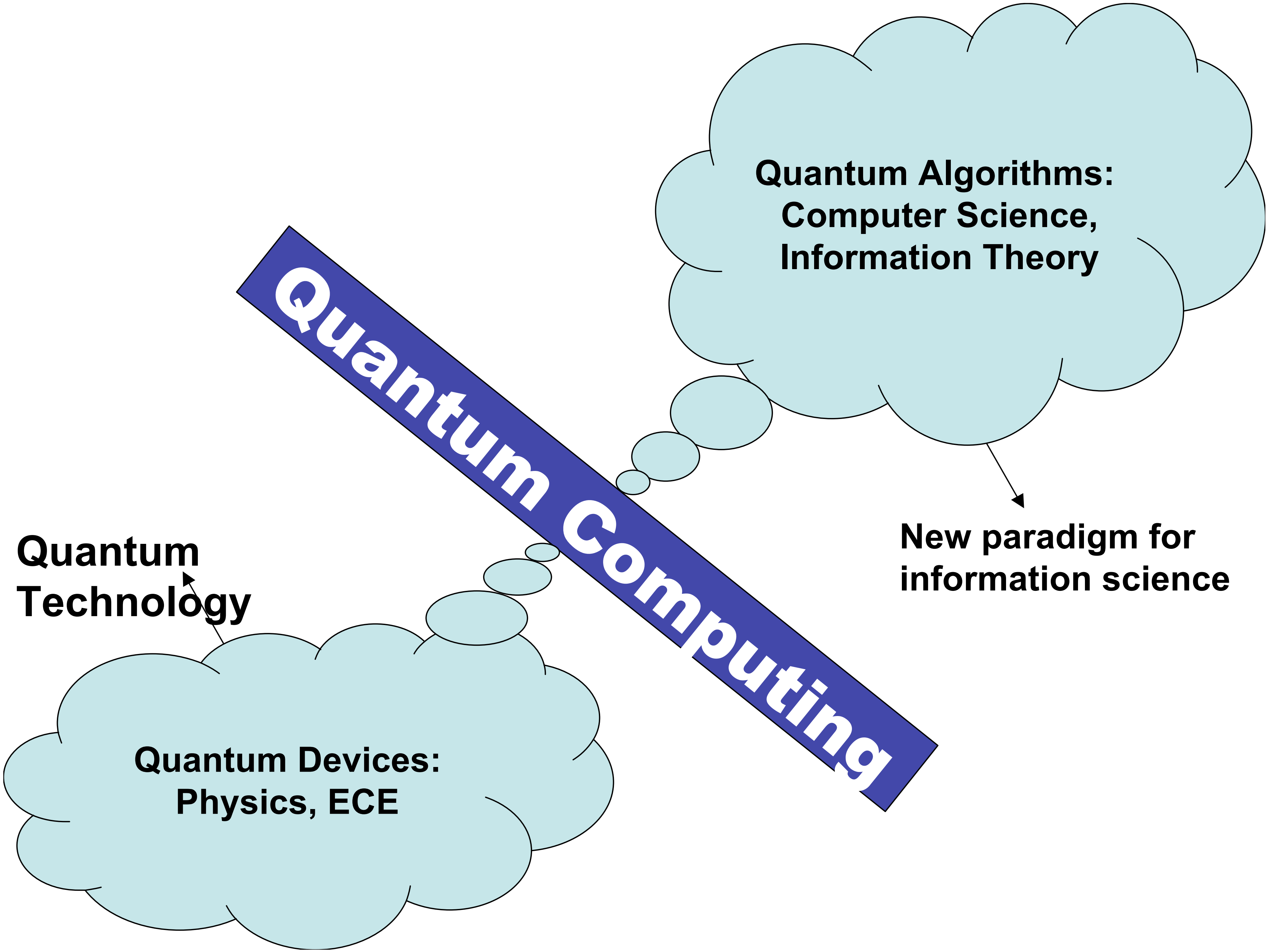
R. Feynman

Charles Bennett

David Deutsch

Peter Shor

1962	1980s	**1994-5**
• Simulate a quantum system with another quantum system?	• First quantum algorithm • Quantum teleportation	**• Prime Factorization** **• Quantum Error Correction**



Quantum Algorithms:
Computer Science,
Information Theory
Quantum Computing
New paradigm for
information science
Quantum
Technology
Quantum Devices:
Physics, ECE

Thinking qubits

Qubit: two level quantum system

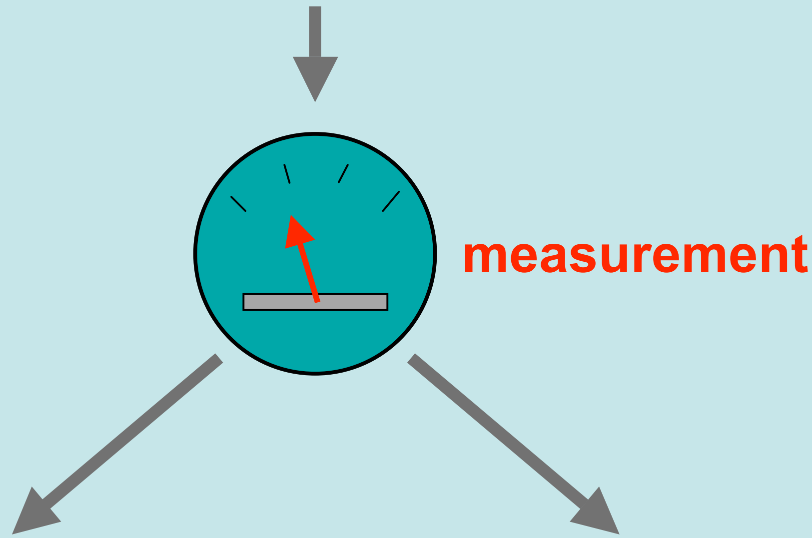
1 classical bit:

$b = 0$ or 1

1 qubit:

$|b\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$

$|R\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$



$|0\rangle$ (probability $|\alpha_0|^2$)

$|1\rangle$ (probability $|\alpha_1|^2$)

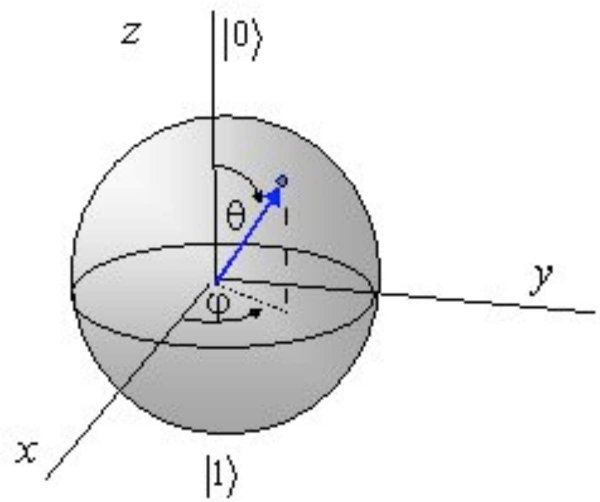
Formalism

Quantum superposition

Qubit: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\longrightarrow$ $|\pm\rangle = \dfrac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}$

“off” **“on”** **“off AND on”**

The Bloch sphere



$$w = w_0|0\rangle + w_1|1\rangle \equiv \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Multiple qubits:

$$|0\rangle \otimes |1\rangle \otimes |0\rangle =$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} =$$

$$\begin{pmatrix} 1\begin{pmatrix}0\\1\end{pmatrix} \\ 0\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = [8 \times 1]$$

dimensional
Hilbert
space

Formalism 2

State vector formalism of quantum mechanics

$$H|\psi\rangle = E|\psi\rangle$$

state evolution $$|\psi\rangle \rightarrow e^{-iH\Delta t/\hbar}|\psi\rangle = U|\psi\rangle$$

Density matrix formalism

$$\rho \rightarrow U\rho U^{t}$$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho]$$

$$\rho_{QC} = \rho_1 \otimes \rho_2 \otimes \rho_3 \otimes \cdots$$

Superposition

- $|R\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ is a ***superposition*** of $|0\rangle$ and $|1\rangle$. $\{\alpha_0, \alpha_1\}$ are the ***amplitudes***.

- **For an *n*-qubit register there are 2^n amplitudes.** ($n = 3$) $\{\alpha_{000}, \alpha_{001}, \alpha_{010}, \alpha_{011}, \alpha_{100}, \alpha_{101}, \alpha_{110}, \alpha_{111}\}$.

- **When $n = 500$, $\{\alpha_{00\ldots0}, \ldots, \alpha_{11\ldots1}\}$ is the size of the universe!**

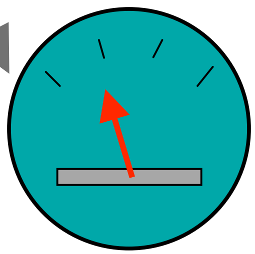
- $|0110001\rangle \xrightarrow{\text{classical operation}} |1101011\rangle$

$\{\alpha_{00\ldots0}, \ldots, \alpha_{11\ldots1}\} \xrightarrow{\text{quantum operation}} \{\beta_{00\ldots0}, \ldots, \beta_{11\ldots1}\}$

Entanglement

Unentangled

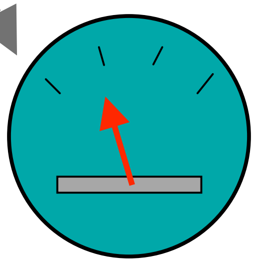
$(|0\rangle + |1\rangle)\times(|0\rangle + |1\rangle)$



qubit 1

$|0\rangle\times(|0\rangle + |1\rangle)$ (prob. 0.5)

$|1\rangle\times(|0\rangle + |1\rangle)$ (prob. 0.5)

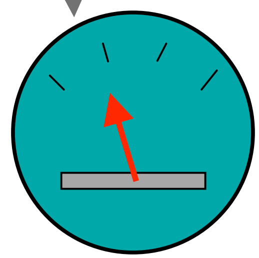


qubit 2

$|00\rangle$ (pr. 0.25) $|01\rangle$ (pr. 0.25) $|10\rangle$ (pr. 0.25) $|11\rangle$ (pr. 0.25)

Entangled

$|01\rangle + |10\rangle$



qubit 1

$|01\rangle$ (pr. 0.5)

$|10\rangle$ (pr. 0.5)

Measurement of qubit 1 fixes state of qubit 2.

Entanglement: Bell States

A quantum state of N qubits that cannot be written as a N-tensor product is said to be entangled.

$$\left|\psi\right\rangle = \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}} \neq \left|?\right\rangle_1 \otimes \left|?\right\rangle_2$$

$$\left|\psi\right\rangle = \frac{\left|01\right\rangle + \left|10\right\rangle}{\sqrt{2}}$$

$$\left|\psi\right\rangle = \frac{\left|00\right\rangle - \left|11\right\rangle}{\sqrt{2}}$$

$$\left|\psi\right\rangle = \frac{\left|01\right\rangle - \left|10\right\rangle}{\sqrt{2}}$$

“Spooky action at a distance”

Building a Quantum Computer

Quantum Algorithms / Computer Science, Math

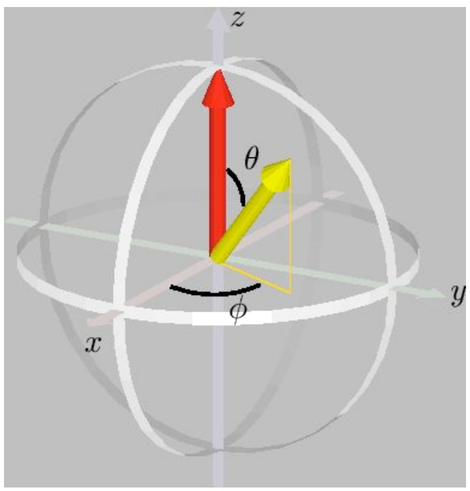
- **Good, scalable qubit**
- **Universal set of gates**
- Fast readout (measurement) of qubit
- Fast initialization / source of new qubits
- Quantum Error Correction
- Flying qubits

Universal Q. Computation

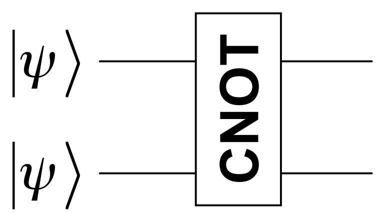
Quantum Algorithms / Computer Science, Math

A *universal* set of gates can compute an arbitrary function (e.g. NAND for classical computation)

Single qubit gates and **CNOT** are a universal set of gates for quantum computation.



$$
\begin{aligned}
\text{cnot}|00\rangle &= |00\rangle \\
\text{cnot}|01\rangle &= |01\rangle \\
\text{cnot}|10\rangle &= |11\rangle \\
\text{cnot}|11\rangle &= |10\rangle.
\end{aligned}
$$



$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

One qubit operations

- **Single qubit rotations on the Bloch sphere**

$$U|\psi\rangle = e^{-iHt/\hbar}|\psi\rangle$$

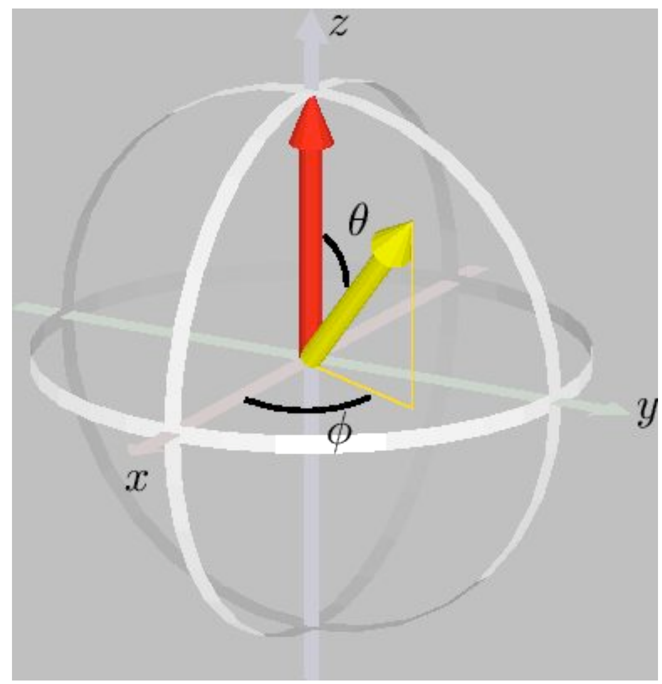
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad -\boxed{\mathbf{X}}- \qquad X|0\rangle = |1\rangle \qquad U = e^{-i\theta X/2}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad -\boxed{\mathbf{Y}}-$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad -\boxed{\mathbf{Z}}-$$

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad -\boxed{\mathbf{H}}- \qquad H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

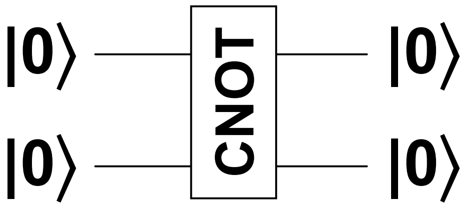
$$T = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\pi/4) \end{pmatrix} \qquad -\boxed{\mathbf{T}}-$$



CNOT

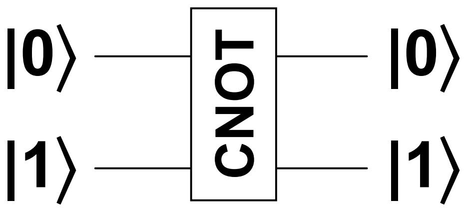
$$
\begin{aligned}
\text{cnot}|00\rangle &= |00\rangle \\
\text{cnot}|01\rangle &= |01\rangle \\
\text{cnot}|10\rangle &= |11\rangle \\
\text{cnot}|11\rangle &= |10\rangle.
\end{aligned}
\qquad
U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

$|0\rangle|0\rangle >>$



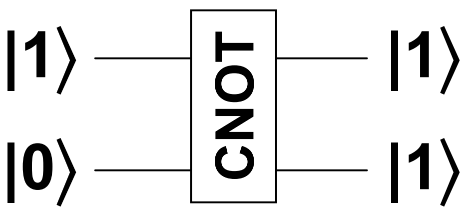
$>>|0\rangle|0\rangle$

$|0\rangle|1\rangle >>$



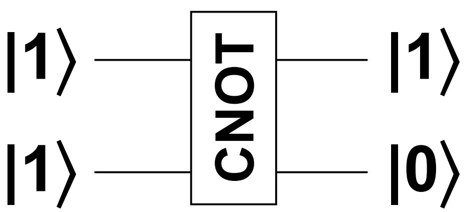
$>>|0\rangle|1\rangle$

$|1\rangle|0\rangle >>$



$>>|1\rangle|1\rangle$

$|1\rangle|1\rangle >>$

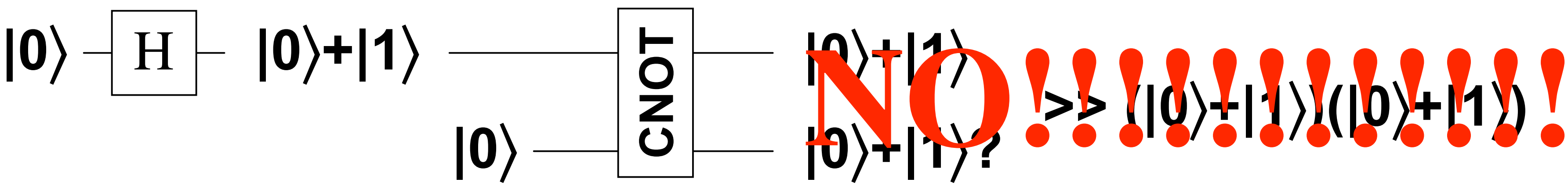


$>>|1\rangle|0\rangle$

CNOT

$$
\begin{aligned}
\mathrm{cnot}|00\rangle &= |00\rangle \\
\mathrm{cnot}|01\rangle &= |01\rangle \\
\mathrm{cnot}|10\rangle &= |11\rangle \\
\mathrm{cnot}|11\rangle &= |10\rangle .
\end{aligned}
$$

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



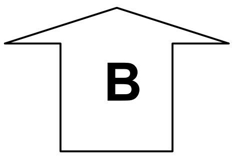
$$(|0\rangle+|1\rangle)|0\rangle = |00\rangle + |10\rangle$$

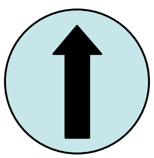
$$\mathrm{CNOT}[\, |00\rangle + |10\rangle \,] = |00\rangle + |11\rangle$$

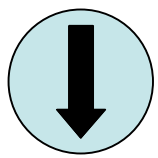
The state cannot be written on two separate lines.

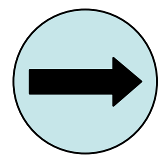
Good qubit: Spin

- Electronic or nuclear spin 1/2
- Natural 2 level system
- Long coherence times
- Scalable (?) in semiconductor structures



 $\rho_{|0\rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

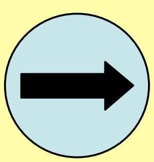
 $\rho_{|1\rangle} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

 $\rho_{|+\rangle} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

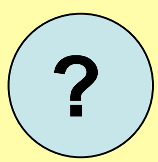
Example:

$T_1 >> T_2$

$t = 0$

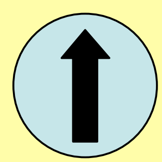
 $\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$T_1 > t > T_2$



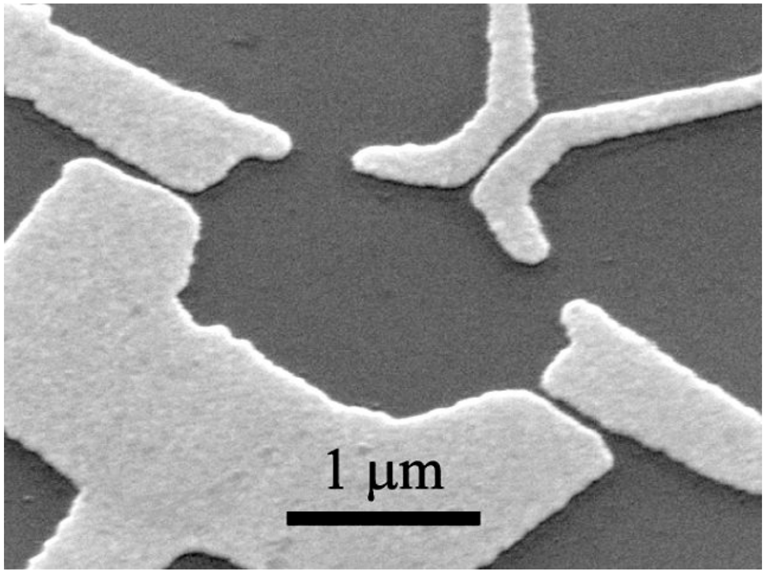
$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$t > T_1$

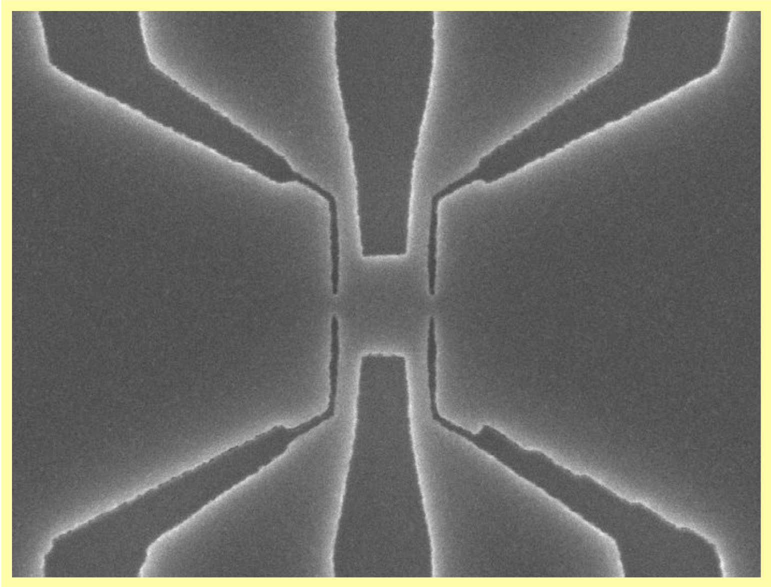
 $\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Quantum Dot Architectures

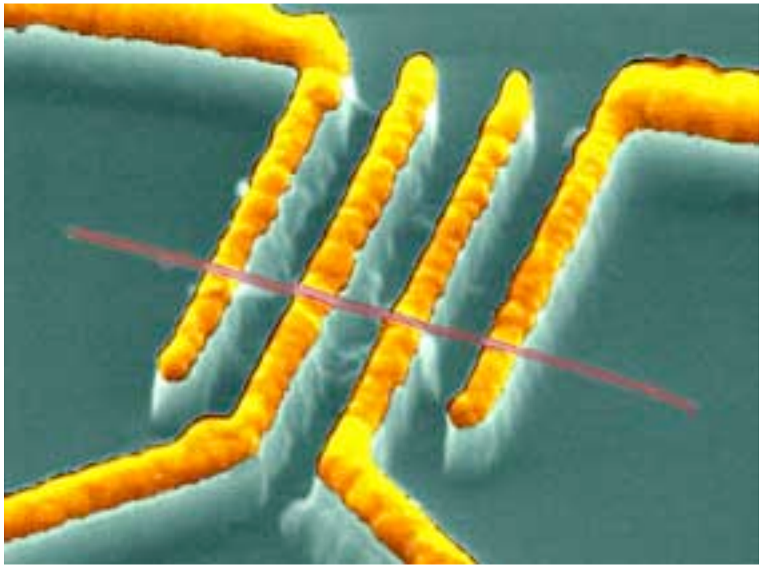
GaAs/
AlGaAs



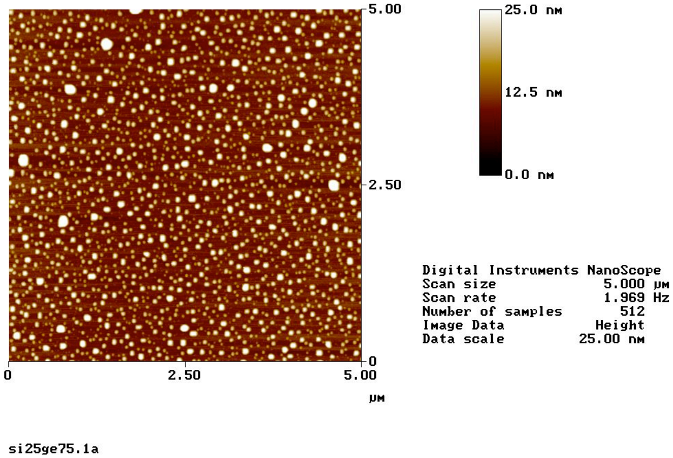
Si/SiGe



Carbon Nanotubes

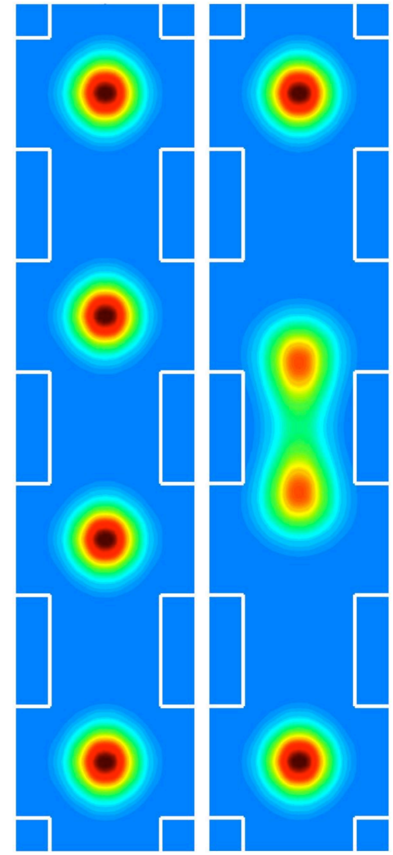


Ge Huts

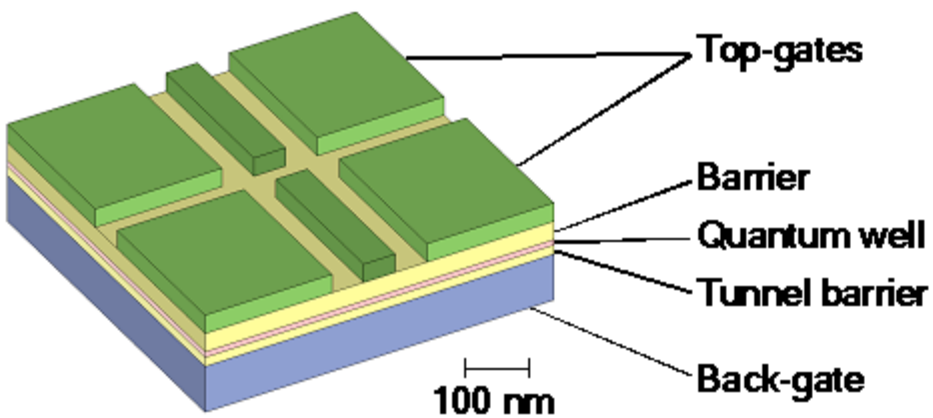


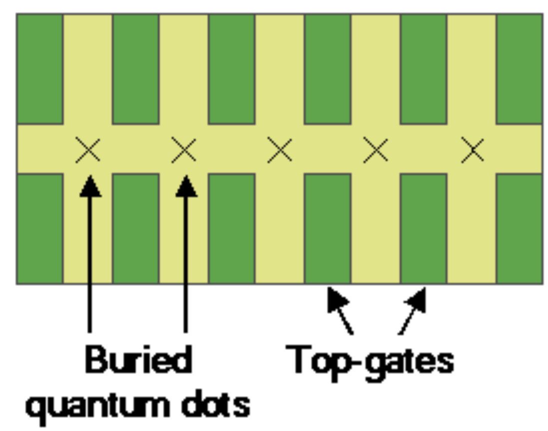
Wisconsin QDQC design

1. **Gated Quantum Dot QC** (Loss & DiVincenzo)
   - 1 electron spin = 1 qubit
   - Self aligning to gates (no need to align to donors)
   - Fast operations through *Heisenberg exchange*
   - Scalable (hopefully)
2. **Silicon**
   - Long decoherence times (T_2 ~ milliseconds for P:^{28}Si)
   - Low spin-orbit coupling
   - Spin-zero nuclei ^{28}Si
3. **Back-gate**
   - Size-independent loading and well-screened manipulation of dots



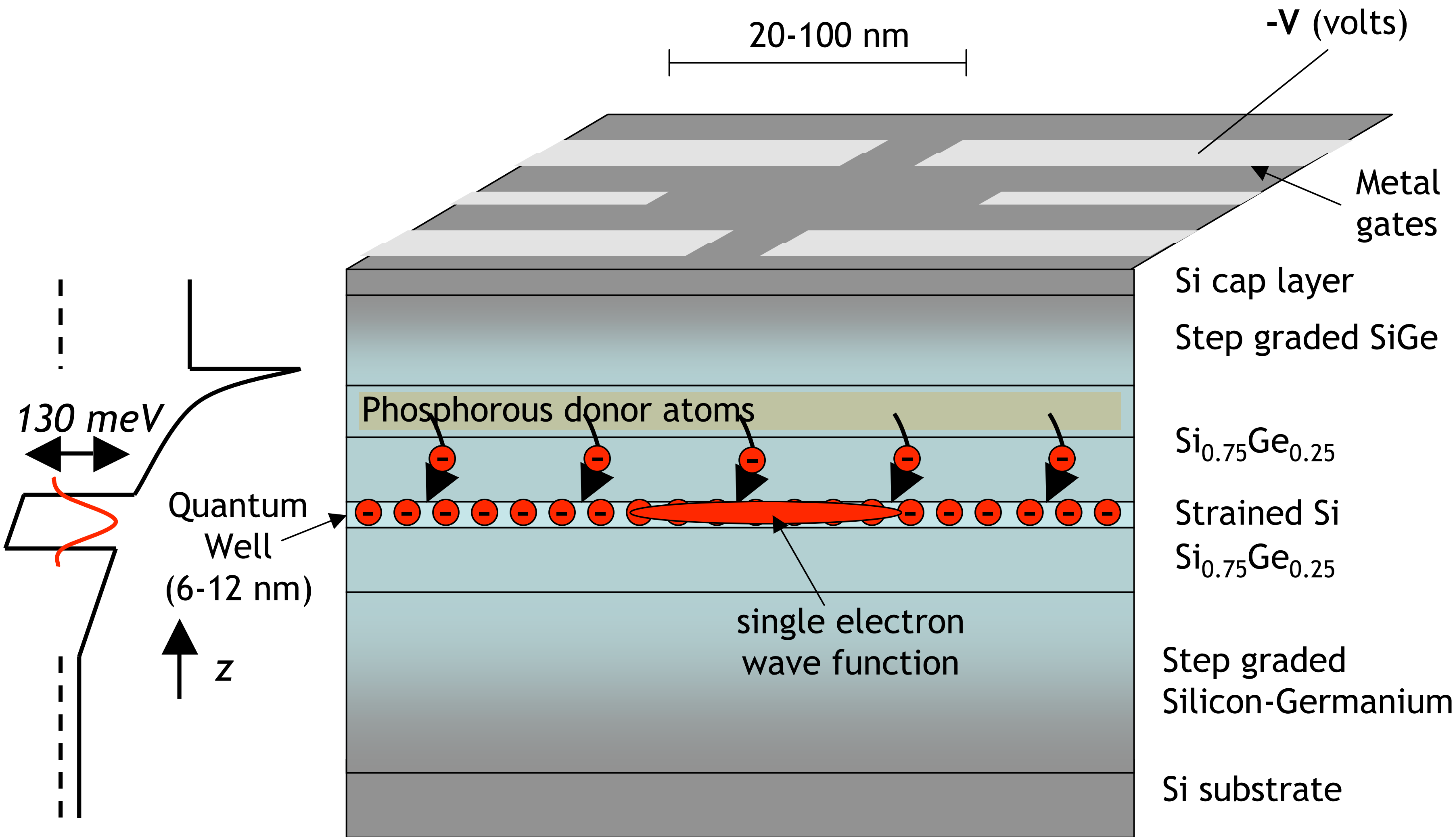
[Freisen, *et.al.*, APL]
[Freisen, *et.al.*, PRB]





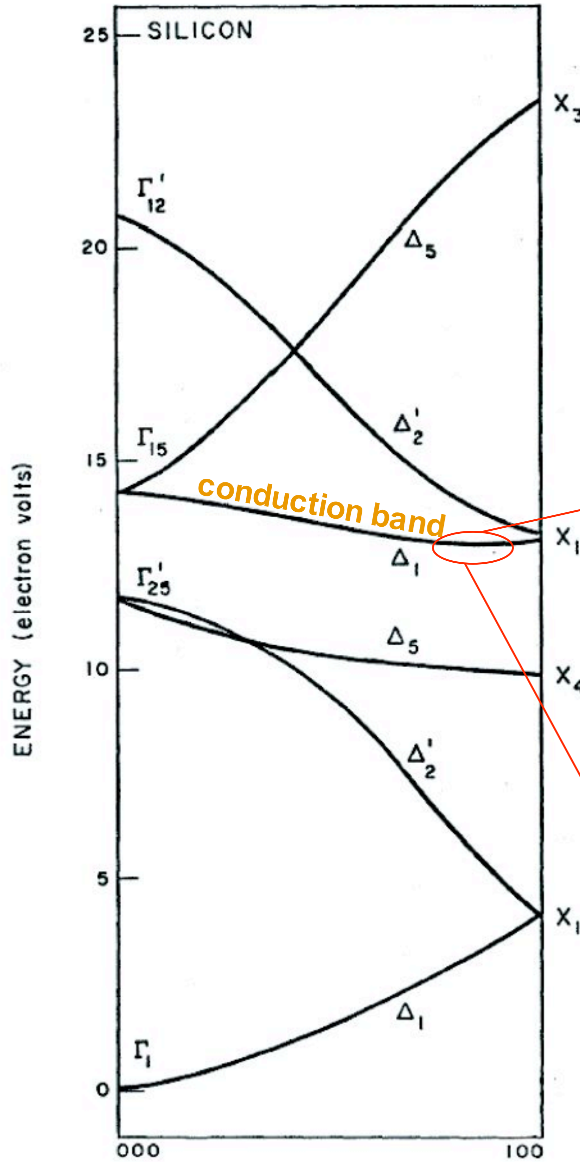
A quantum well quantum dot

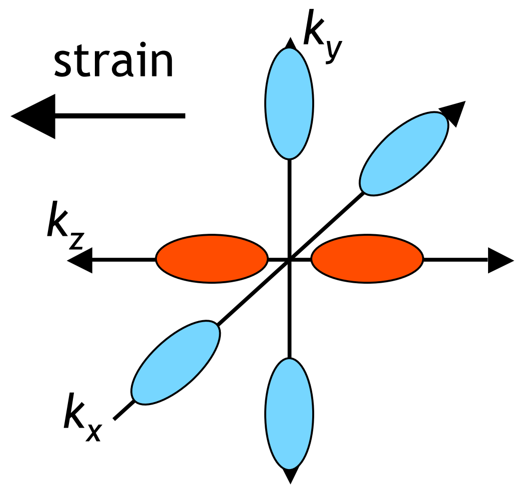
Goal: a single electron tunably confined vertically and horizontally in a semiconductor nanostructure



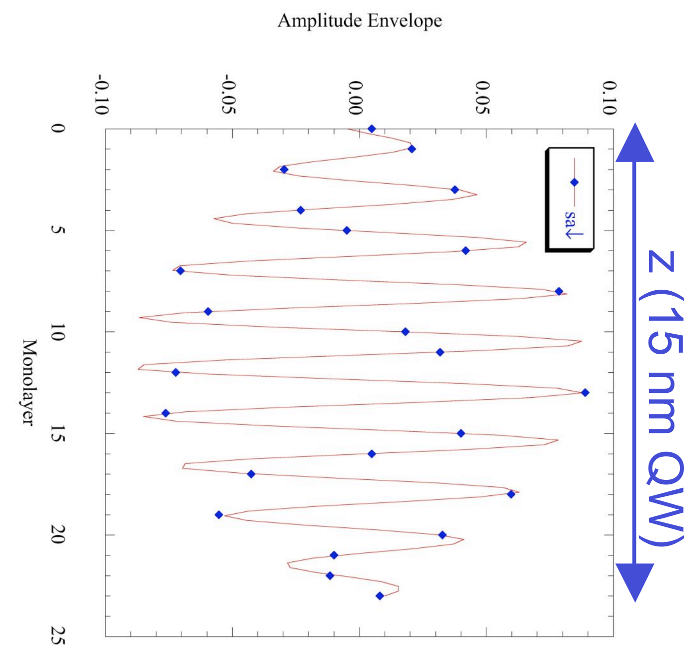
Details...

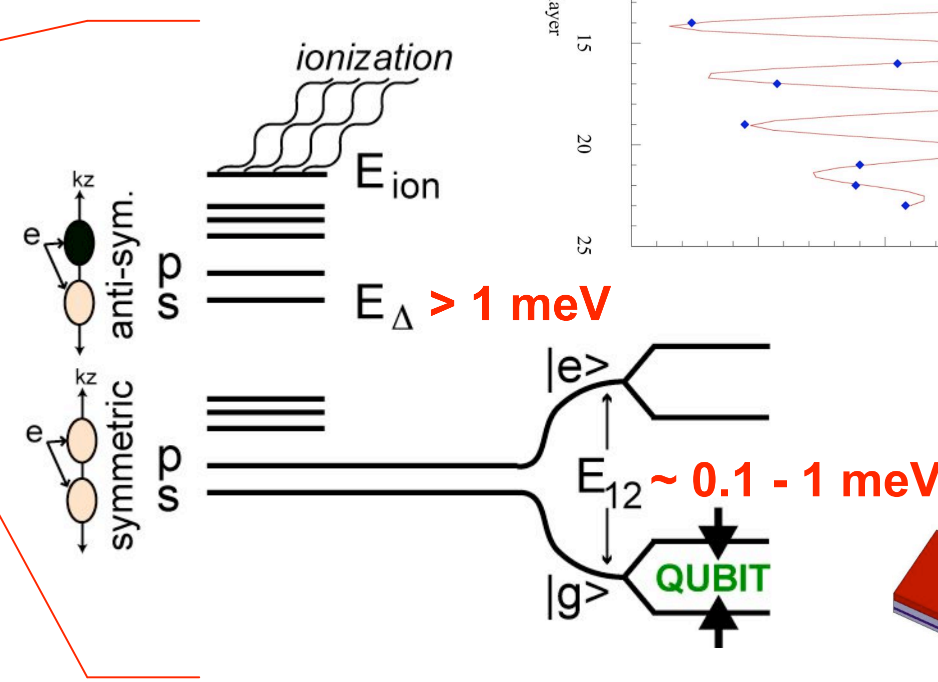
$$|\Psi\rangle = \text{Envelope} \times \text{Bloch}$$

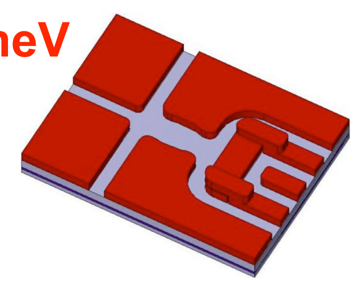




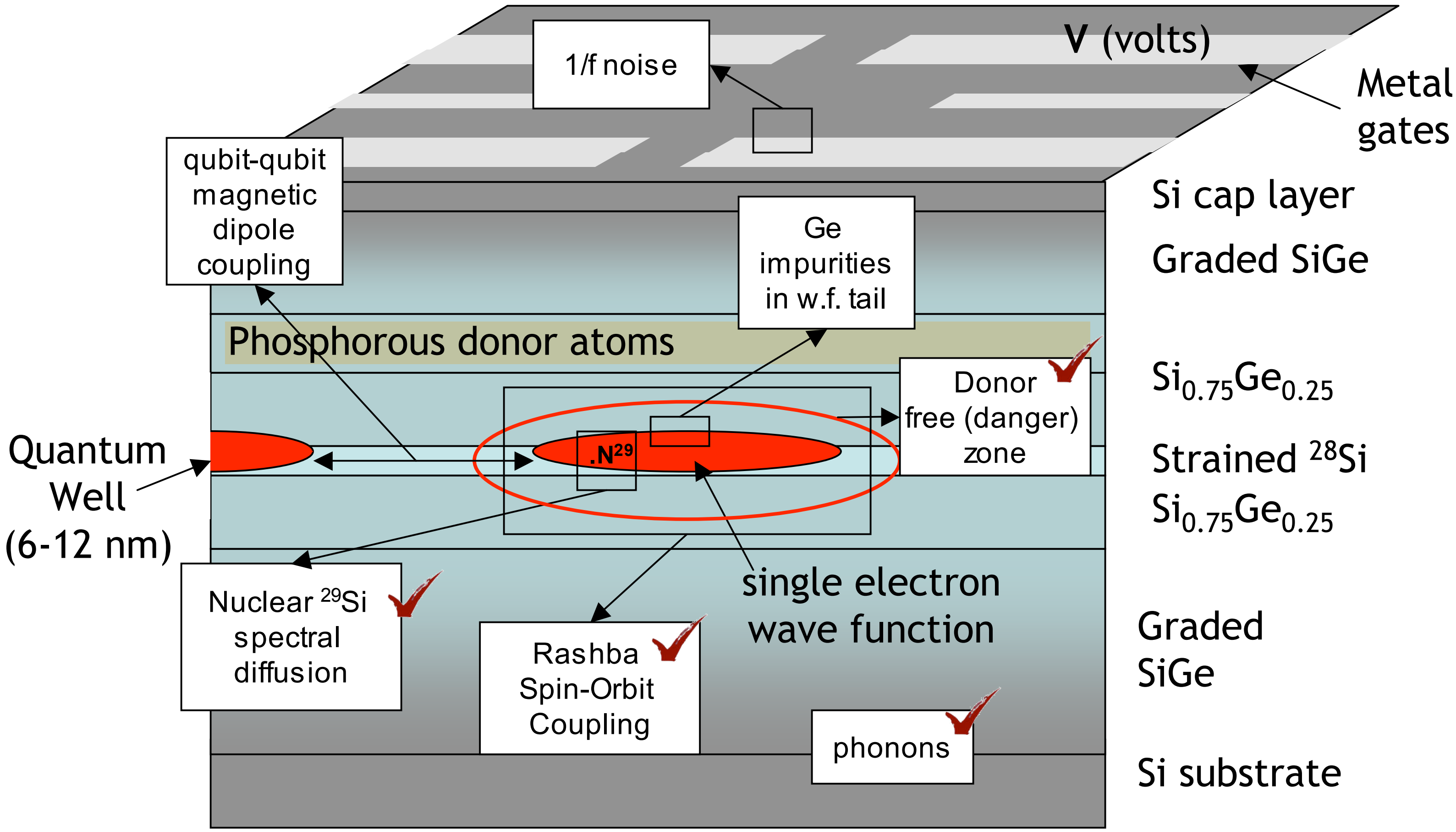
Bloch functions





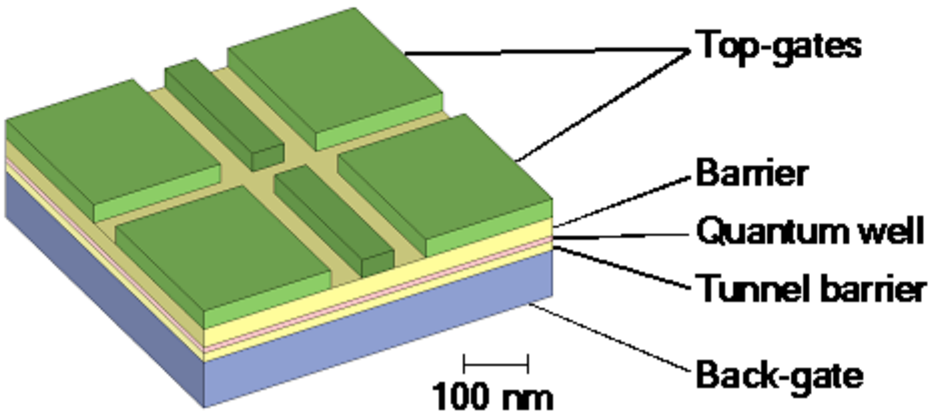


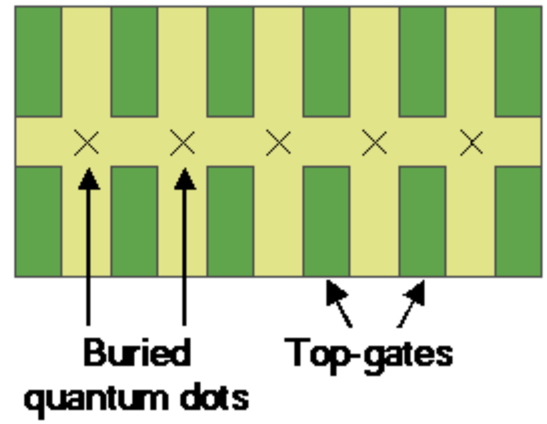
Decoherence

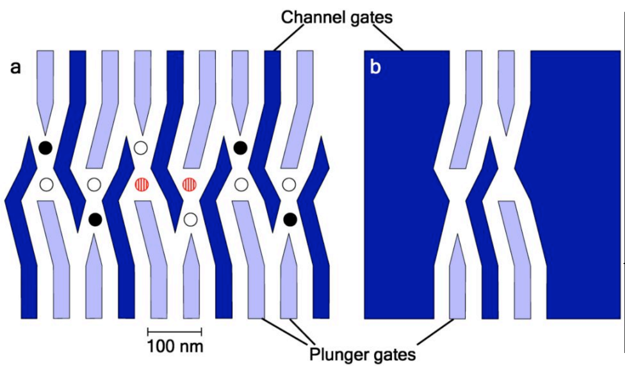


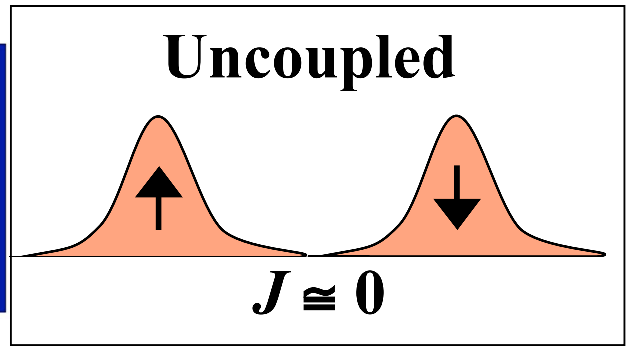
$T_1 \sim$ milliseconds $\sim T_2$

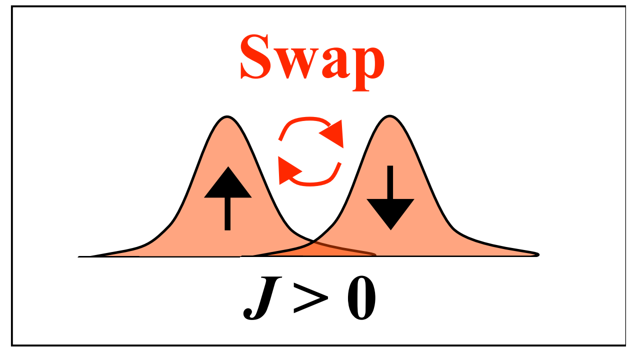
Exchange and CNOT











$$\mathrm{H}_{2\ \text{quantum dots}} \longrightarrow \mathrm{H}_{\text{eff}} = J\ \boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

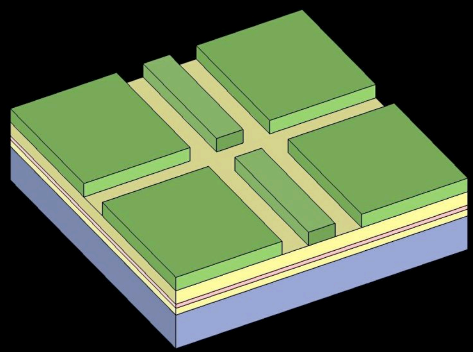
SWAP doesn't entangle but Sqrt[SWAP] does.

SWAP: $\mathrm{Int}[J(t)\ dt] = \pi$

=> CNOT

Simulation: Coupled Qubits in Silicon

(Friesen, Rugheimer, Savage, et al., ’03)



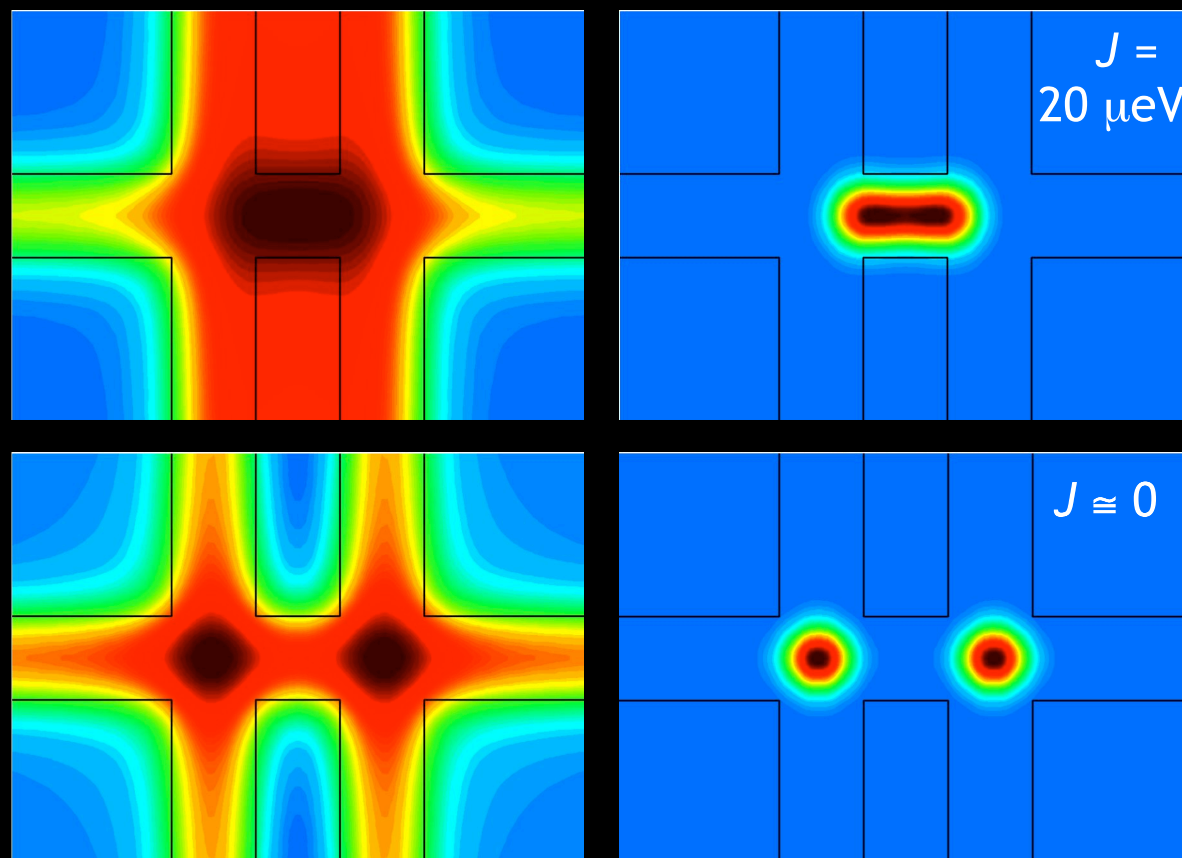
$$|S\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$|T\rangle = \begin{cases} |\uparrow\uparrow\rangle \\ \dfrac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \\ |\downarrow\downarrow\rangle \end{cases}$$

$$J = E_S - E_T$$

on

off



screened potential

probability density

unscreened potential

Exchange and CNOT

$$H_{\text{eff}} = J(t)\, \boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

$$U(t)|\psi\rangle = e^{\frac{i}{\hbar}\mathbf{S}_1 \cdot \mathbf{S}_2 \int J(t)dt}$$

$$\mathbf{S}^2 = \mathbf{S}_1{}^2 + \mathbf{S}_2{}^2 + 2\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$\int J(t)dt = JT/\hbar = \theta$$

$$U = e^{i\theta \mathbf{S}_1 \cdot \mathbf{S}_2} = \exp\left[i\theta \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] = e^{i\theta/4} U_{SWAP}$$

$$U_{CNOT} = e^{i(\pi/2)Z_1} e^{-i(\pi/2)Z_2} \sqrt{U_{SWAP}}\, e^{i(\pi/2)Z_1} \sqrt{U_{SWAP}}$$

Readout and Initialization

- Reading out a single spin is hard!

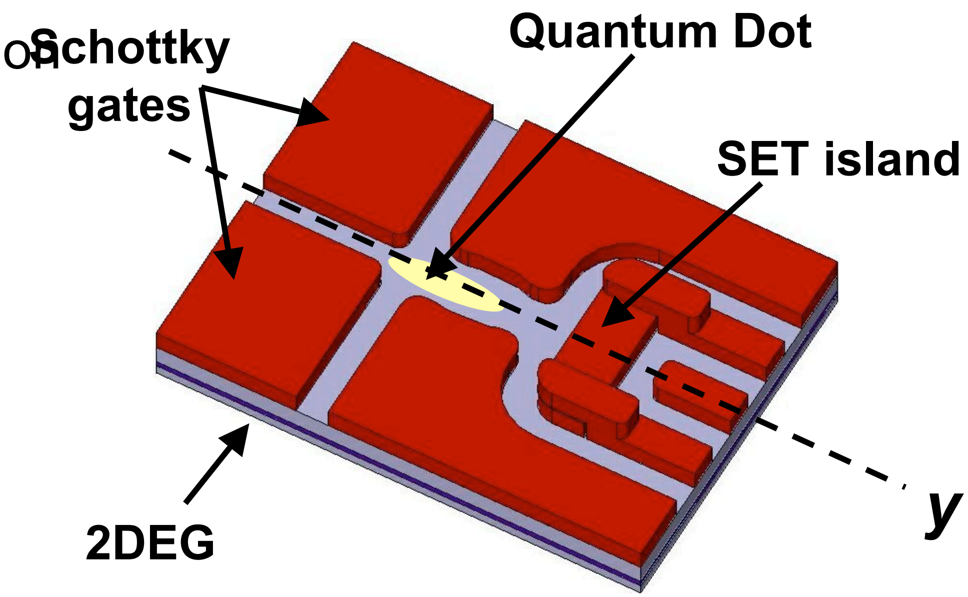
$$\mathbf{m}_e = 9.3 \times 10^{-24} \text{ J/T}$$
$$\mathbf{m}_{refrig} = 0.1 \text{ J/T}$$

- Magnetic STM tip
- Spin-charge transduction
  – Spin-blockade transport measurement
  – Spin-orbital transduction

Device design for QD readout

[Friesen, Tahan, Joynt, Eriksson, *Phys. Rev. Lett.*]

- Spin-dependent charge motion
- SET detection
- Microwave pumping
- Automatic spin polarization

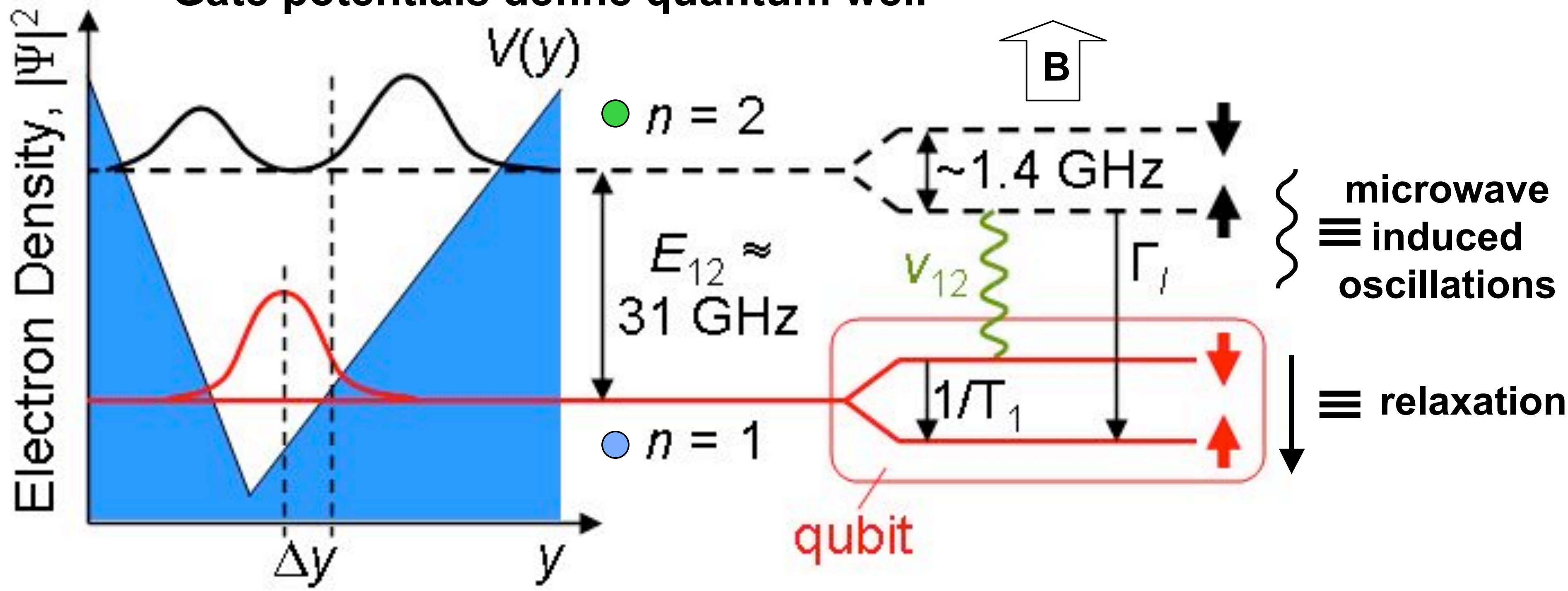


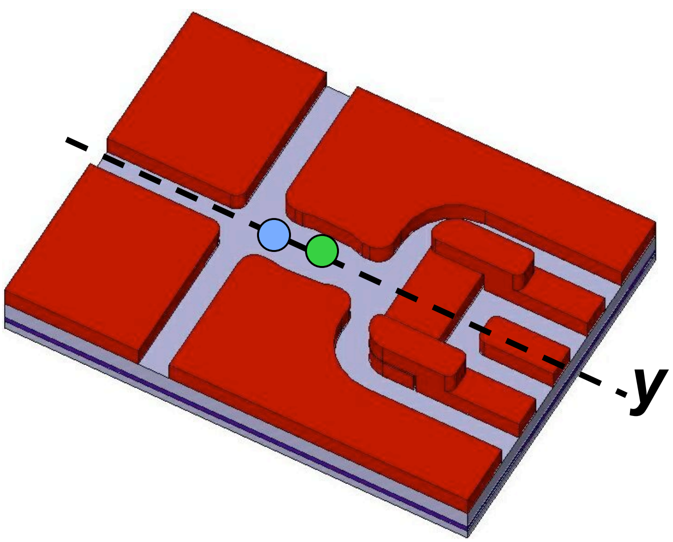
Fast readout and initialization is important for error correction

History…spin-charge transduction Loss/Divincenzo, Kane, …

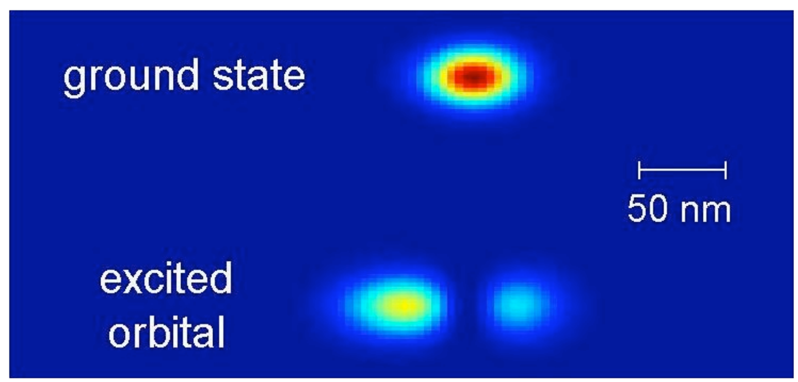
Charge movement in asymmetric well

Gate potentials define quantum well





• spin info to charge info via spin-dependent excitation



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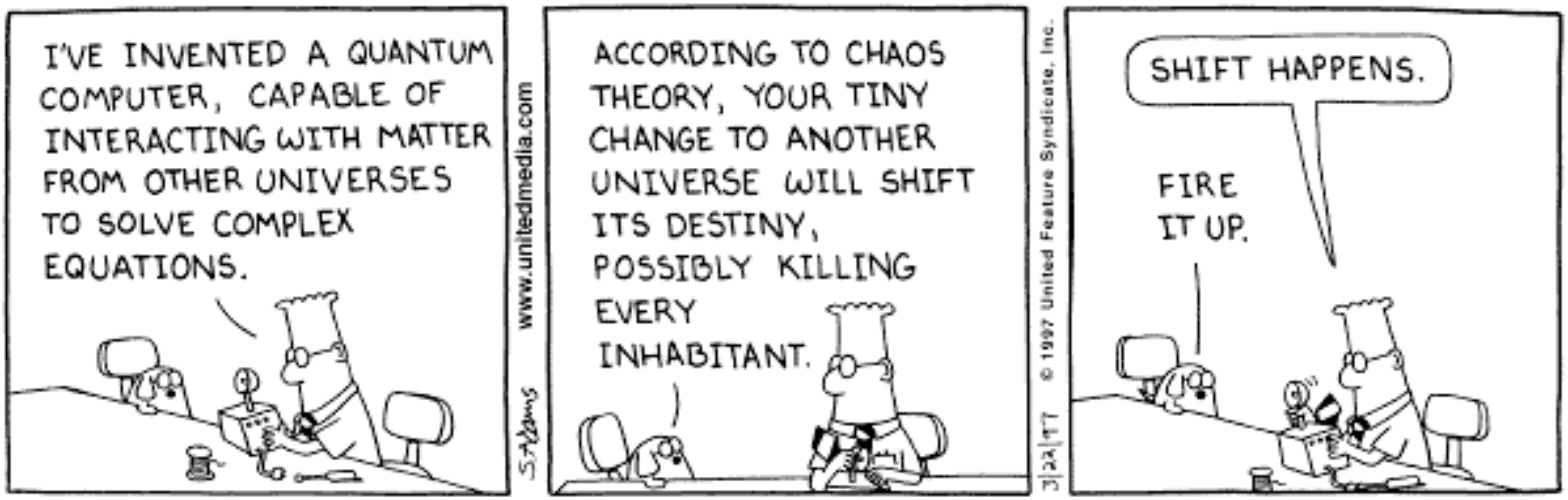
Intermission



Quantum Power

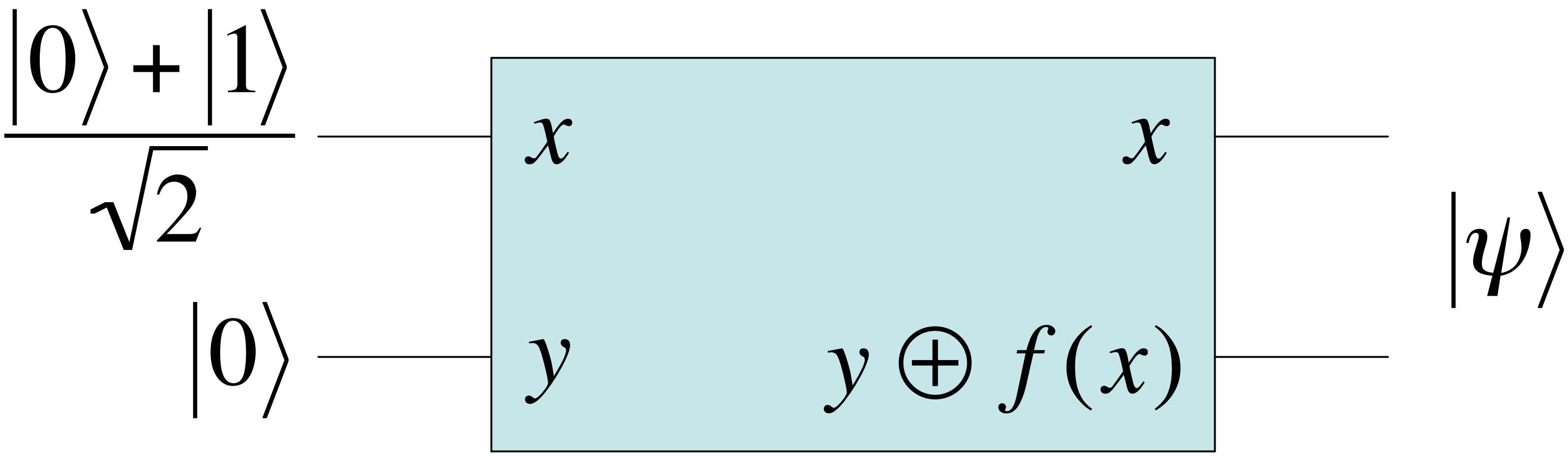
Review

- Superposition (and large Hilbert space)
- Entanglement
- Interference (waves)

But we need to ask the right questions.

Quantum Parallelism

f(x) is a binary function: $f(\{0,1\}) = \{0,1\}$

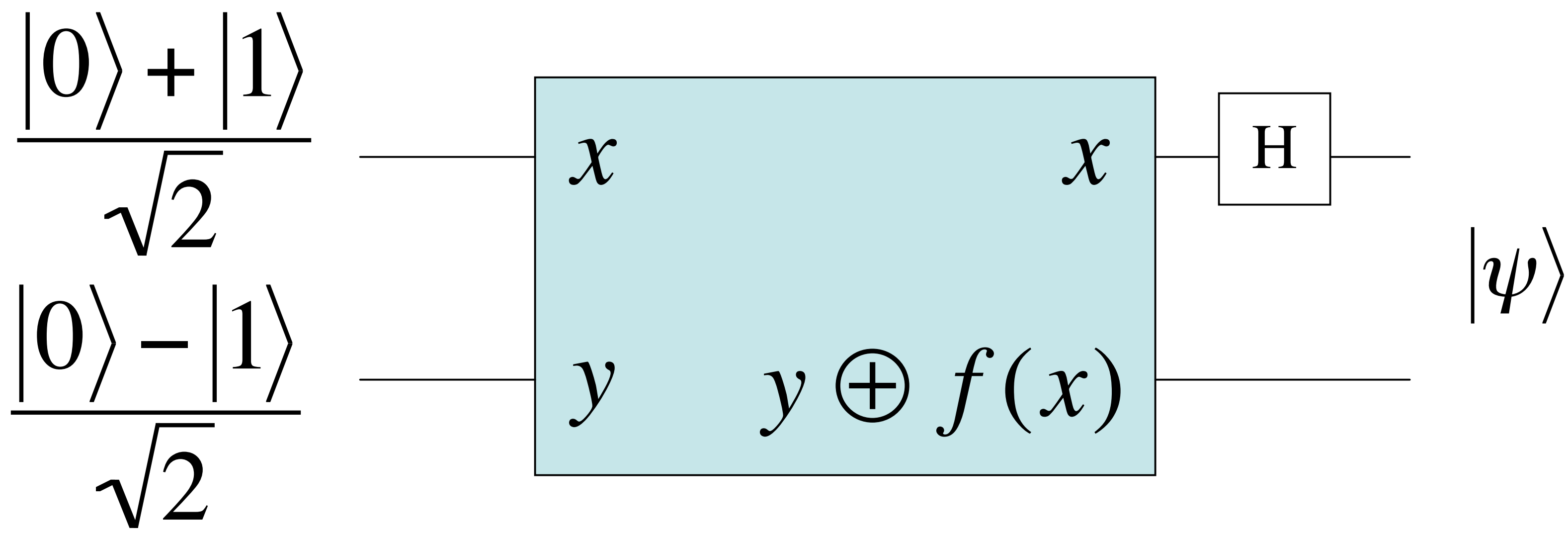


$$|\psi\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

Measurement will only choose one!

Deutsch Algorithm

Ask a global question: Is the function f(x) constant or not?

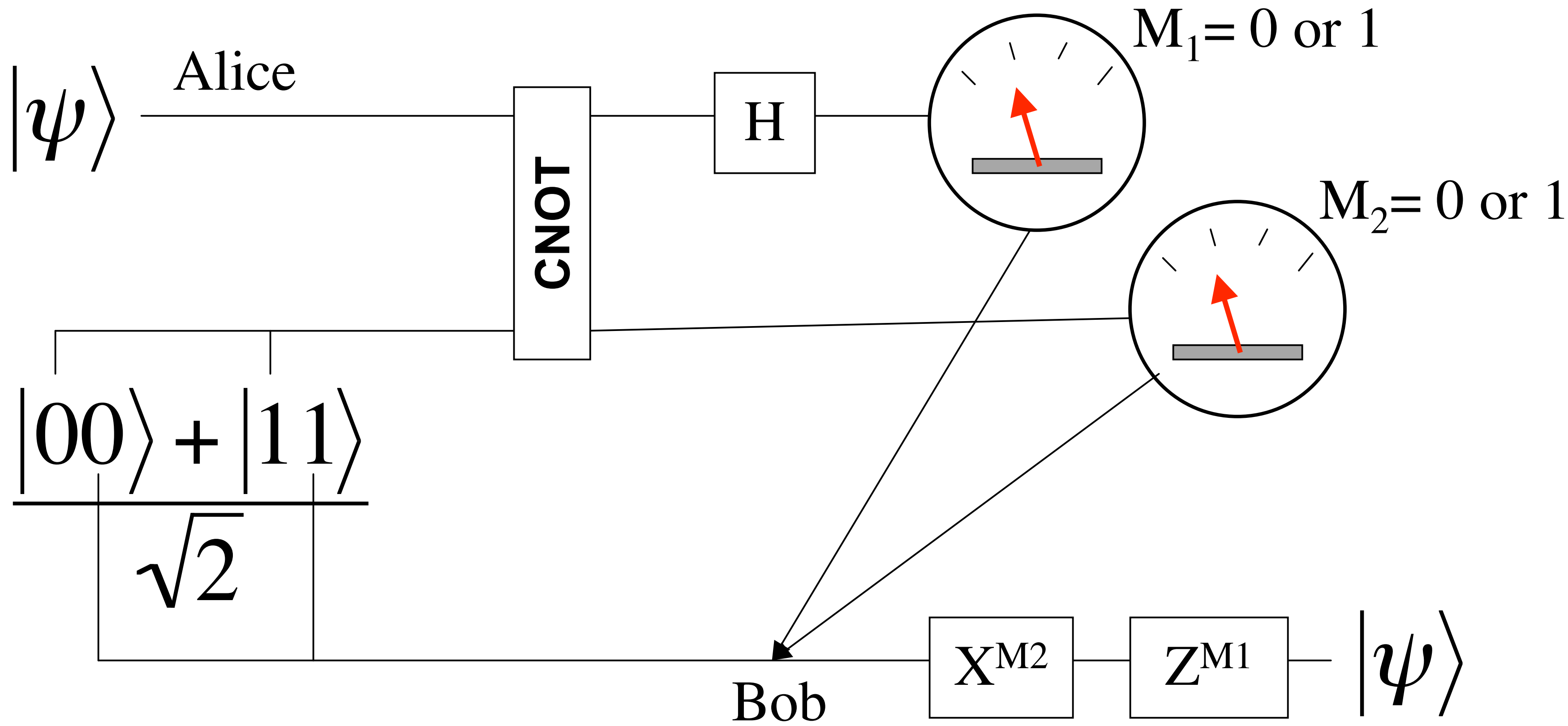


$$|\psi\rangle = \pm\left| f(0) \oplus f(1) \right\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]$$

Qubit 1 encodes the answer to the global question.

Quantum Teleportation

Want to send the state psi from alice to bob.

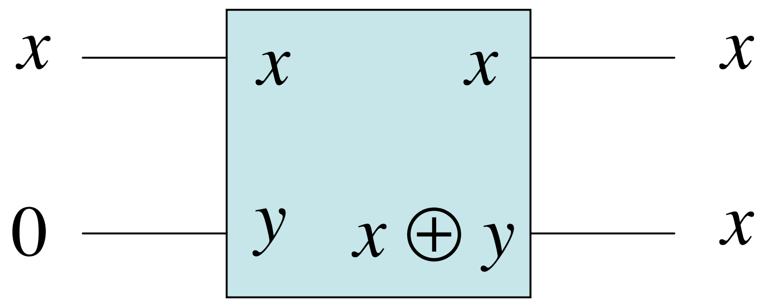


The qubit state is transferred from Alice to Bob utilizing the entanglement of the Bell state as a resource. It is not copied.

No cloning theorem

No cloning theorem: it is NOT possible to make a copy of an unknown quantum state

Classical copying circuit:

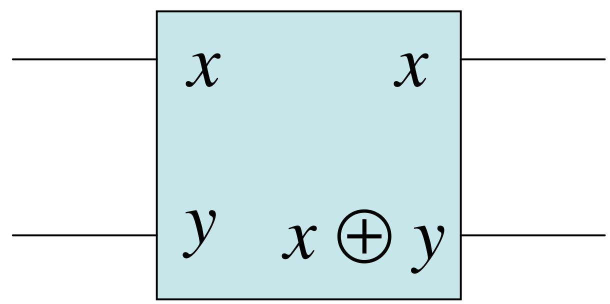


$$\Rightarrow xx$$

Quantum version

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$$|0\rangle$$



$$\Rightarrow a|00\rangle + b|11\rangle$$

But, $|\psi\rangle|\psi\rangle = a^2|00\rangle + ab|01\rangle + ab|10\rangle + b^2|11\rangle$

QEC - Active

No cloning theorem: it is NOT possible to make a copy of an unknown quantum state

The Shor code: 9 qubits

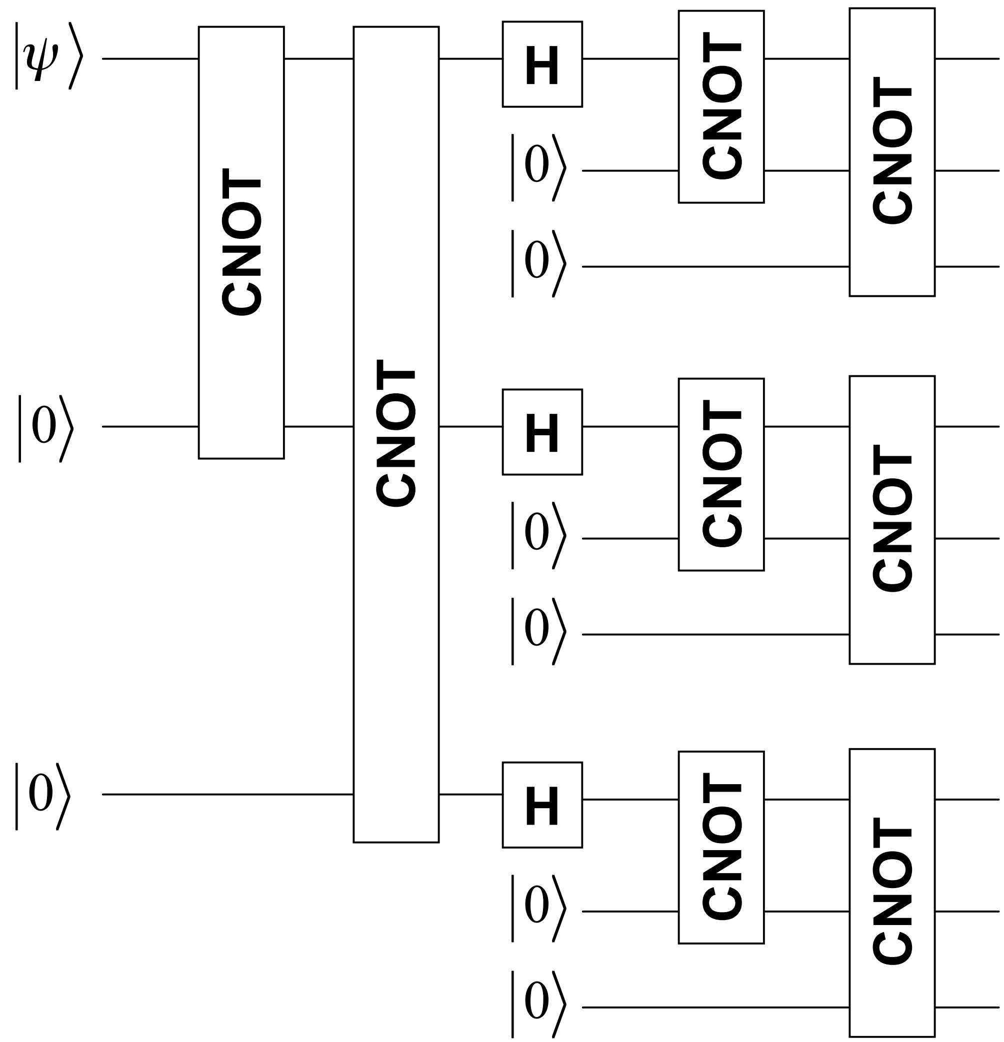
phase flip code

$$|0\rangle \rightarrow |0\rangle_L = |+++\rangle$$
$$|1\rangle \rightarrow |1\rangle_L = |---\rangle$$

bit flip code

$$|0\rangle \rightarrow |0\rangle_L = |000\rangle$$
$$|1\rangle \rightarrow |1\rangle_L = |111\rangle$$

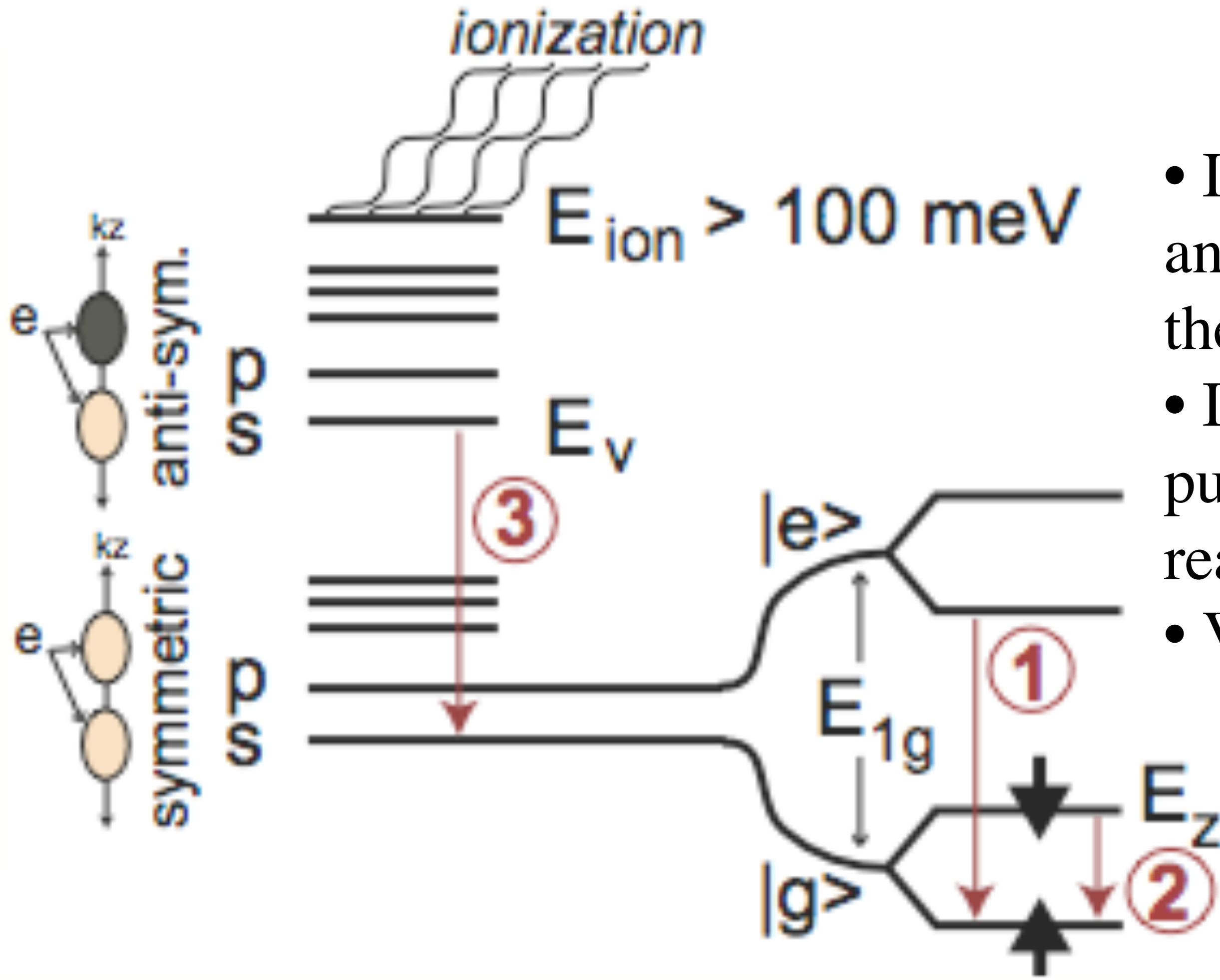
Threshold theorem



Nuts and Bolts stuff:

How long is decoherence

Relaxation in a Si quantum dot



- If no spin-1/2 nuclei and phonons dominate, then $T_1 \sim T_2$
- Important optical pumping and our readout scheme
- Valley effects

New theory results for silicon

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1. **2DEG spin relaxation**: *Correct anisotropy and magnitudes. Times will increase with <u>mobility and B</u>.*

2. **Rashba Coefficient**: $\alpha \approx 1 - 6 \text{ m/s}$

3. **T_1 in Si quantum dots**: *Rashba SOC usually dominates. Time increases with <u>smaller dots</u> and <u>smaller B-fields</u>.*

4. **Valley-state lifetimes**: *Microseconds to milliseconds. Long-lived pseudo-spin states.*

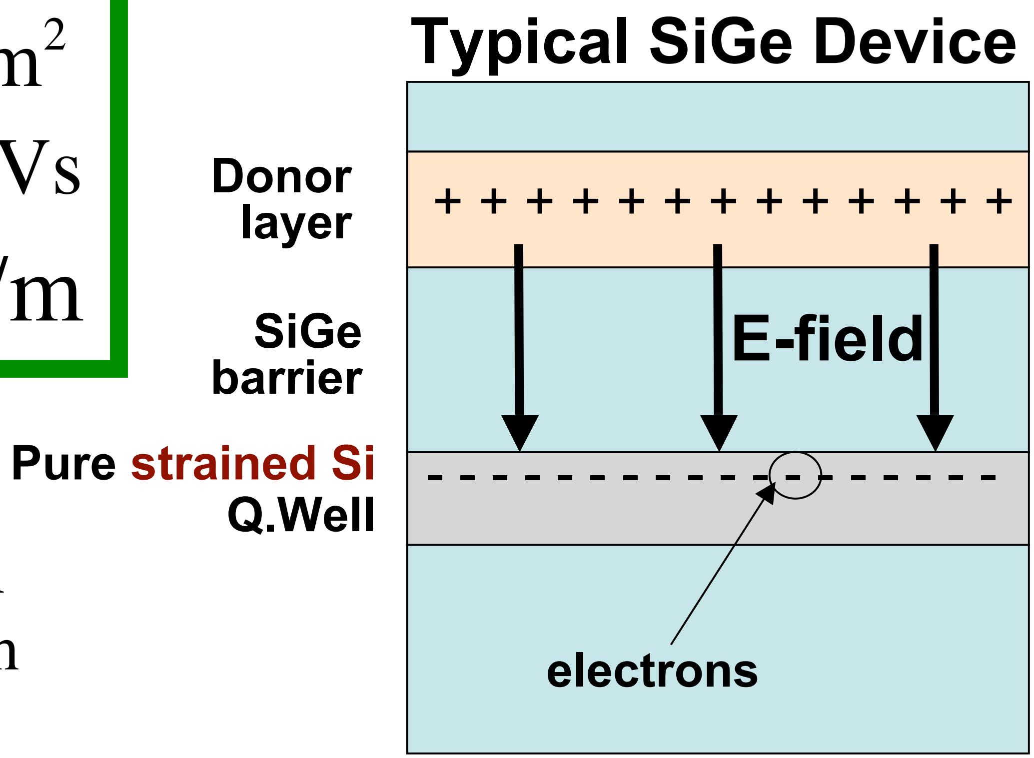
SiGe quantum wells for QDQC

qc.physics.wisc.edu

Our device:

$$n_s = 4 \times 10^{11}\ \text{e/cm}^2$$
$$\mu = 40{,}000\ \text{cm}^2/\text{Vs}$$
$$E = 6 \times 10^6\ \text{V/m}$$

- Experiments: CW-ESR of SiGe 2DEGS as a path toward dots (also for spintronics).



Friesen, *et. al.*, PRB **67**, 121301R (2003)

Spin relaxation in SiGe 2DEGs

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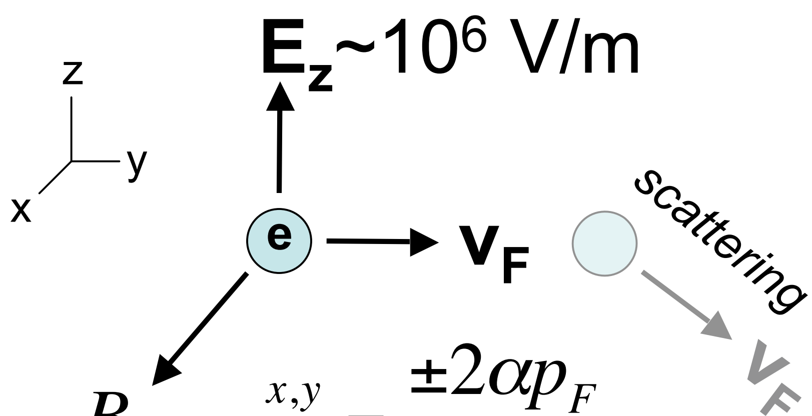
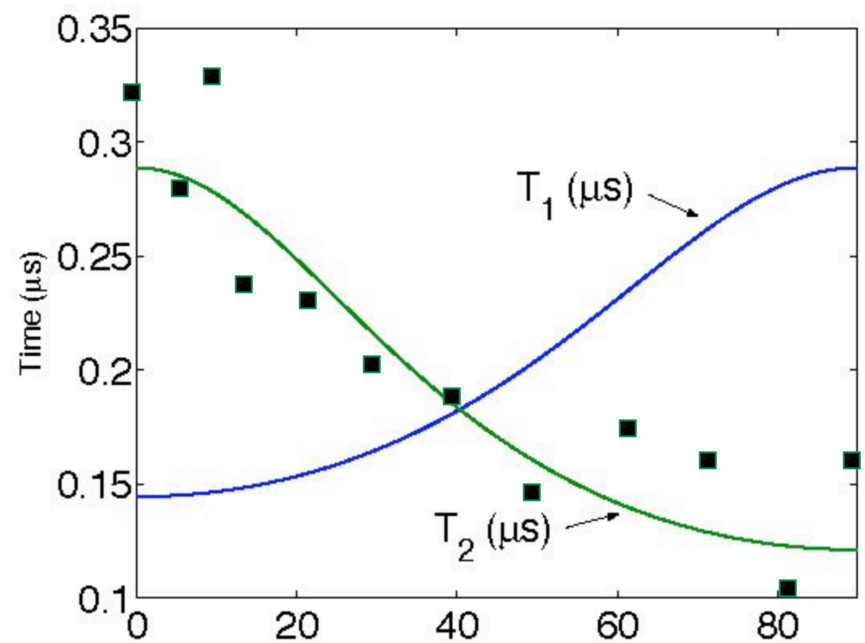
- Rashba SOC + scattering = fluctuating B-field in-plane
- Anisotropic T_1 and T_2

T ~ 5 K



$$B_{Rashba}^{x,y} = \frac{\pm 2\alpha p_F}{g\mu_B}$$



Magnetic field angle from z

$$\frac{1}{T_2(\theta)} = \left[2\alpha^2 p_F^2 \tau_p \sin^2\theta + \frac{\alpha^2 p_F^2 \tau_p}{1+\left(\omega_L \tau_p\right)^2}\left(\cos^2\theta + 1\right)\right] / \hbar^2$$

Tahan and Joynt, condmat/0401615

T_1 & T_2 increase with mobility

…in high mobility limit

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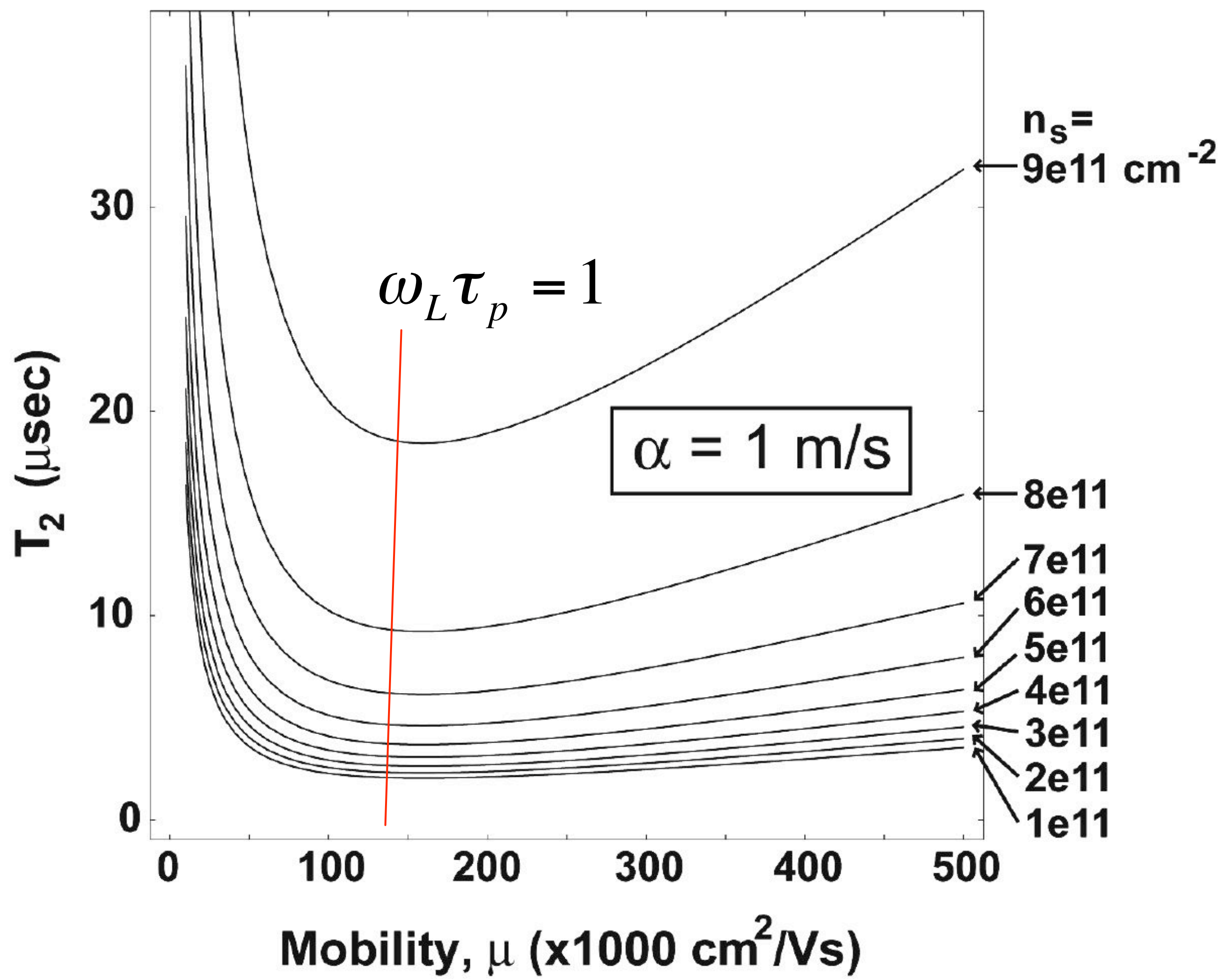
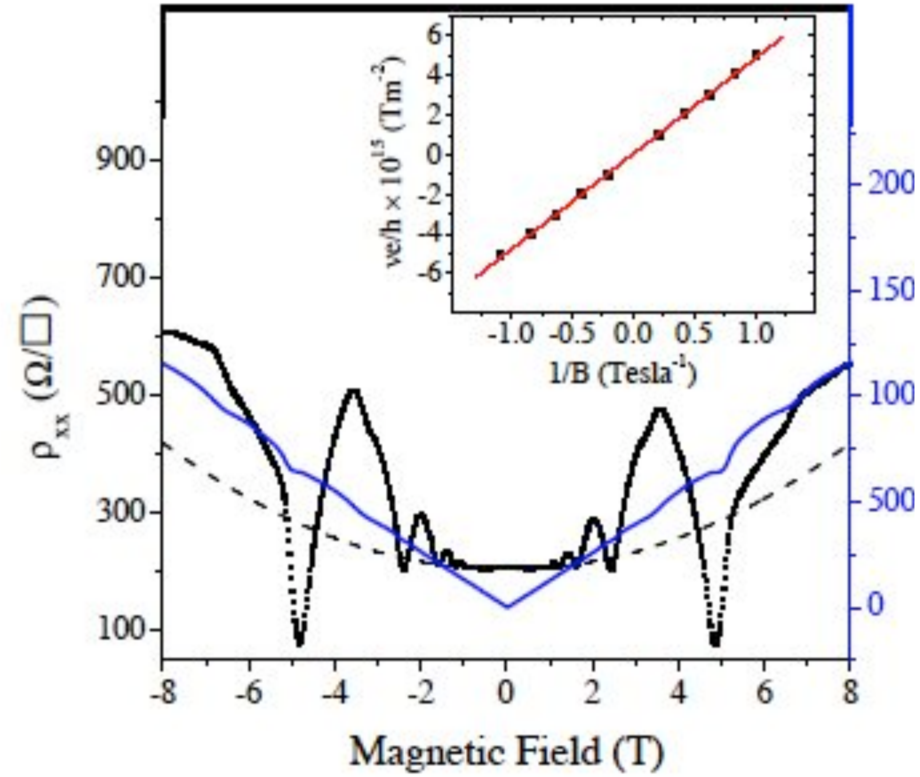
- **In a static B-field**

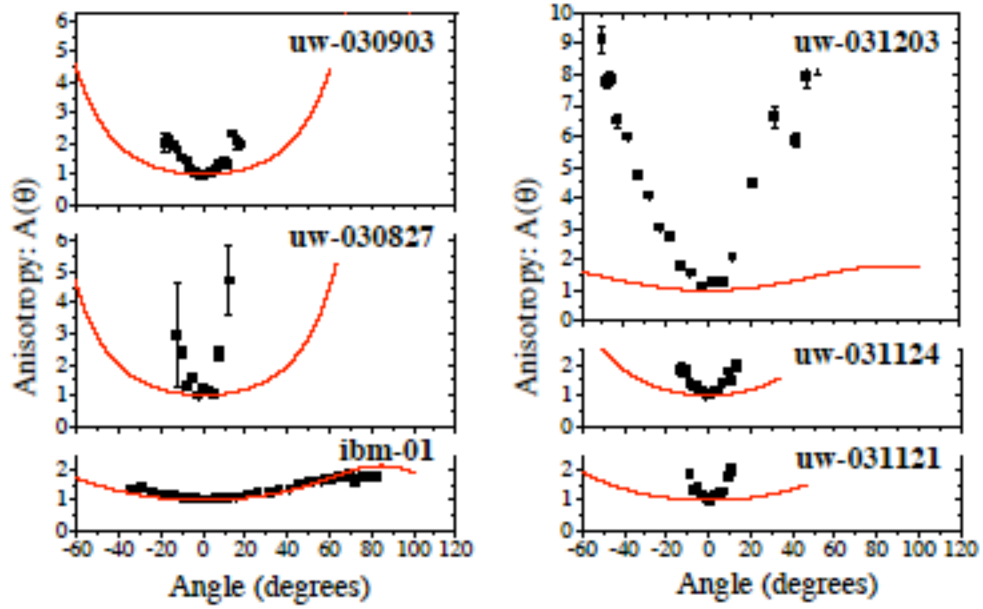
$$\omega_L = \frac{g\mu B}{\hbar}$$



Tahan and Joynt, condmat/0401615

2DEG CW-ESR and transport data





Sample	Si well (nm)	x	offset (nm)	dopant (nm)	spacer (nm)	cap (nm)	n_e ($\times 10^{11}$ cm^{-2})	μ (cm^2/Vs)	τ_P (ps)
ibm-01	8.0	0.30	14	1	14	3.5	4.0	37,300	4.3
uw-030827	10	0.35	15	22	35	10	4.8	90,000	9.7
uw-030903	10	0.25	13	17	35	10	4.3	86,700	9.4
uw-031121	10	0.30	20	6	60	20	5.4	38,000	5.0
uw-031124	10	0.30	20	26	40	20	4.7	63,200	6.9
uw-031203	10	0.30	60	6	60	20	2.6	17,100	1.8

Summary of transport and ESR measurements

	n_e (cm^{-2})	μ (cm^2/Vs)	g-factor	Peak width	τ_p (ps)	α_{RB}
IBM 1	4.0e11	40,000	2.0015	0.14 G	9.2	1.4
UW 030827	4.8e11	90,000	2.0013	0.97 G	109.7	5.6
UW 030903	4.5e11	87,000	2.0005	0.40 G	73.7	2.9
UW 031121	5.1e11	46,000	2.0013	0.71 G	100.0	5.3
UW 031124	4.7e11	64,000	2.0012	0.78 G	77.3	4.4
UW 031203	2.6e11	17,000	2.0003	0.23 G	54.4	1.6

Many more samples grown at UW over the past year – this is the subset that has undergone careful characterization.

Rashba coefficient in silicon

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- **Spin-orbit coupling (SOC) due to large E_z**

$$H_{Rashba} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) \qquad \text{(just from Dirac SOC)}$$

- **Kane-like 8 band calculation for Si**

$$\alpha(\mathbf{E}_z) \approx \frac{2PP_z\Delta_d}{\sqrt{2}\hbar E_{v1}E_{v2}}\left(\frac{1}{E_{v1}} + \frac{1}{E_{v2}}\right)e\langle\mathbf{E}_z\rangle$$

Our theory: $\alpha \approx 1 - 6\ \text{m/s}$

Rashba spin-splitting: $\Delta_R \sim 1\ \mu\text{eV}$

Rashba field: $B_R^{\ x,y} = 10 - 40\ \text{G}$

In-line with experimental evidence.

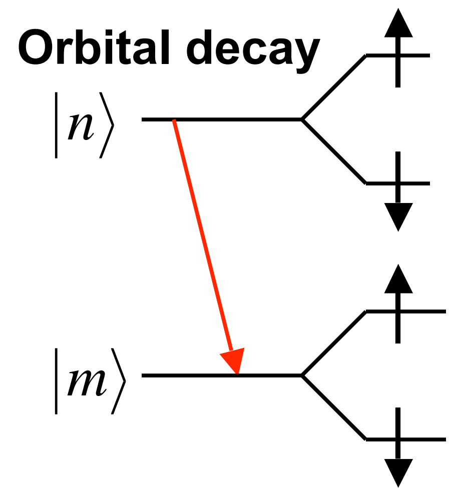
Tahan and Joynt, condmat/0401615

Orbital relaxation in strained Si

- **Deformation interaction (no piezo-phonons in Si)**
- **Strained Si => transverse phonons contribute**

$$\Gamma_{mn}\left(\mathrm{s}^{-1}\right) \approx \frac{E^{5}{}_{mn}}{\hbar^{6}\pi\rho_{Si}}\left|\left\langle m|z|n\right\rangle\right|^{2}\frac{2\Xi_{u}{}^{2}}{105 v_{t}{}^{7}}$$

transverse phonons: $v_t = v_l/2$



- **Speed comparable to GaAs**

$$\boxed{\Gamma_{mn}\left(\mathrm{s}^{-1}\right) \approx \text{ns to ps} \quad \left(\hbar\omega_0 = 0.1\text{ - }1\ \mathrm{meV}\right)}$$

Relevant to:
- **Optical pumping/ Initialization**
- **Many-phonon processes**

T_1 from Rashba in Si QDs

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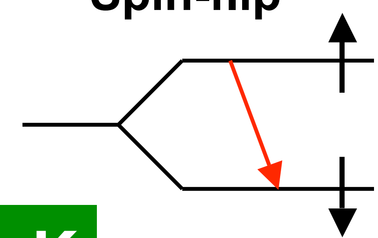
- **Confined state:** $\langle \mathbf{p}_{x,y} \rangle << p_F^{2DEG}$ **=> 2DEG mechanism gone**
- **Rashba spin-orbit mixing + phonon = relaxation**

unexpected: **GaAs** and **bulk Si:** $\sim (g\mu B)^5$

$$\frac{1}{T_1} \approx \frac{\alpha^2 (g\mu B)^7}{\rho \pi \hbar^4 (\hbar\omega_0)^4} \frac{\Xi_u^2}{105 v_t^7} \left(3 + \cos[2\theta]\right)$$

Dot size dependent

Spin-flip



T < 100 mK

- **Non-zero for all directions of B**

T_1	$B = 0.33$ T	$B = 2$ T
$\hbar\omega_0 = 0.1$ meV	5 s	0.002 s
$\hbar\omega_0 = 1$ meV	56,000 s	24 s

Orbach spin relaxation

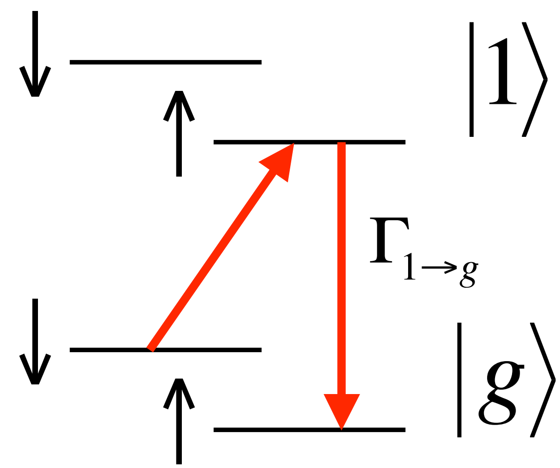
- **Two phonon process**
- **Dominant mechanism in P:Si for T > 4 K**
- **Provides limited spectroscopy of first orbital energy gap**

Relaxation rate of level |1>

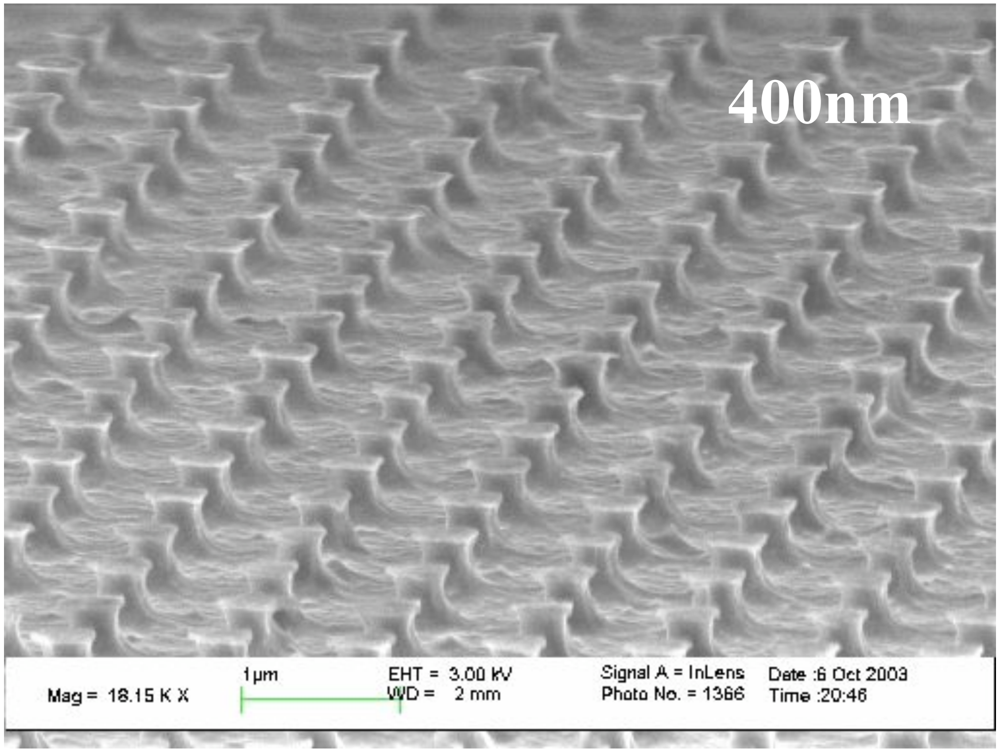
$$\frac{1}{T_1} \approx M^2{}_{SO}\Gamma_{1\to g}e^{\left[-E_{1g}/kT\right]}$$

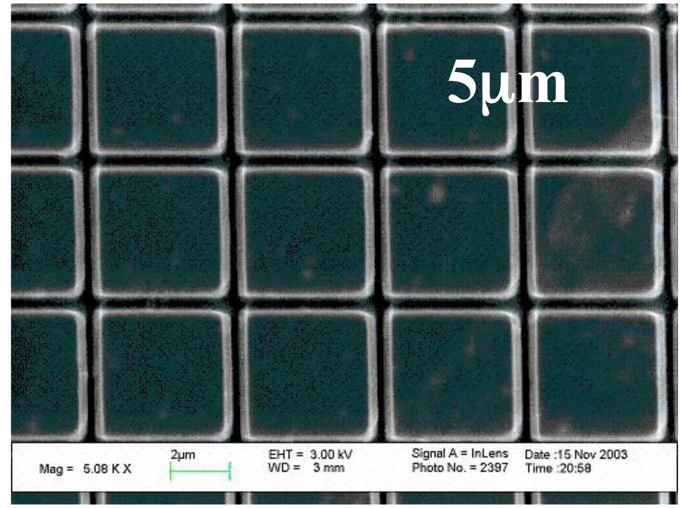
Phonon DOS

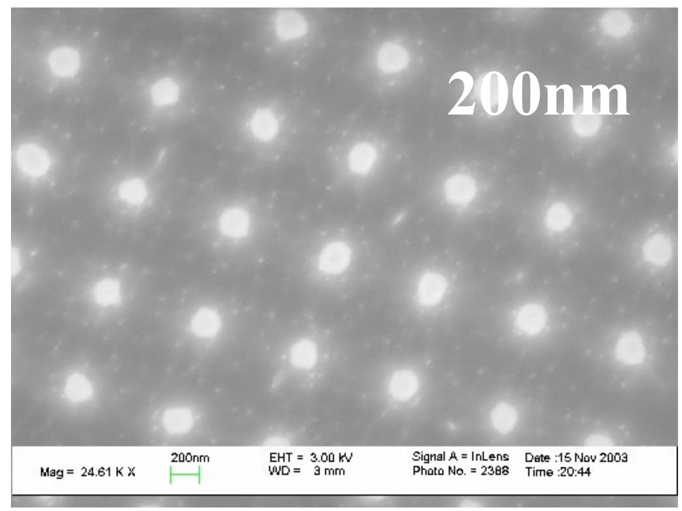
SO mixing
between
|1> and |g>



Etched dots (SEMs)







Dot sizes from 5µm to 200nm fabricated – most are still in many electron limit (i.e., 2DEG-like)

Valley states in silicon QWQDs

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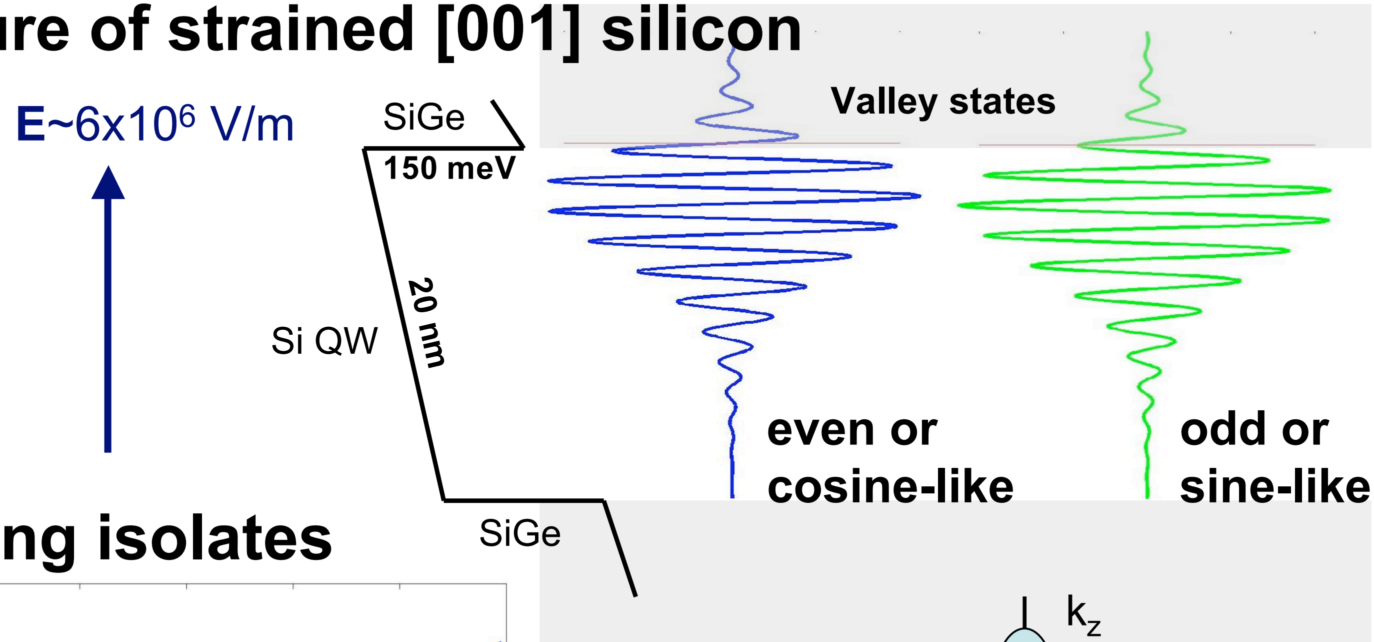
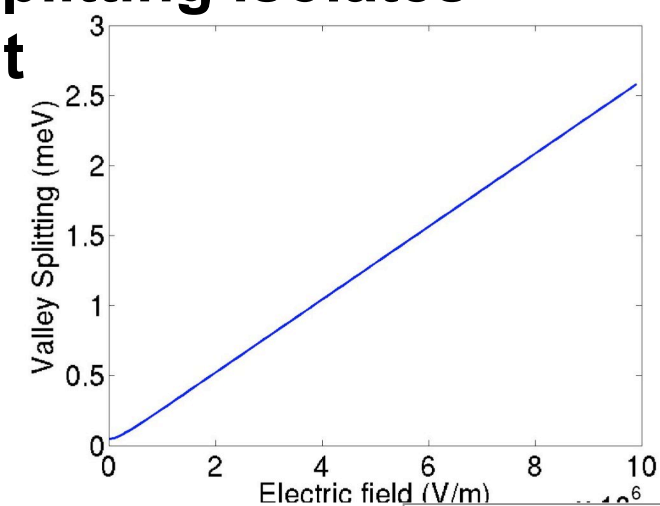
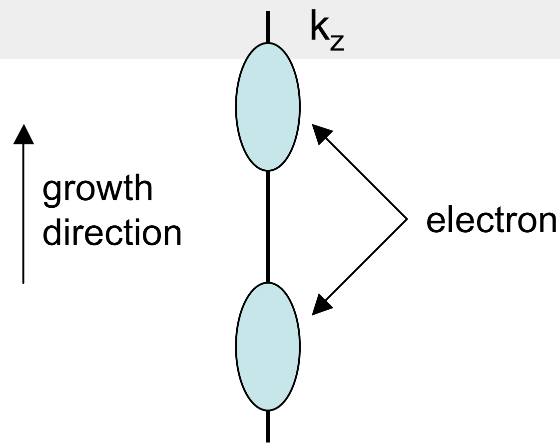
• 2-valley nature of strained [001] silicon



• Valley-splitting isolates spin qubit





Boykin, Klimeck, Coppersmith, *et. al.*, APL **84**, 115 (2003)

Valley-state lifetimes

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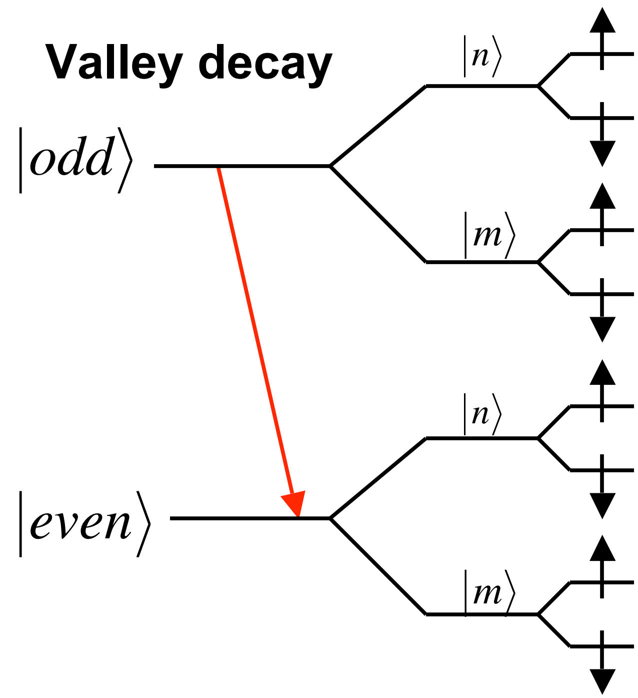
- **Same procedure as orbital relaxation**
- **Extremely small electric-dipole matrix element**

$$\boxed{\Gamma^{v}{}_{mn}\left(\mathrm{s}^{-1}\right) \approx \text{microsecs - millisecs}}$$



- **Long relaxation times for non-spin-flip transitions**
- **Tunable with external E-field**