Madison, WI

Spin-flip transitions in silicon quantum dots

Charles Tahan

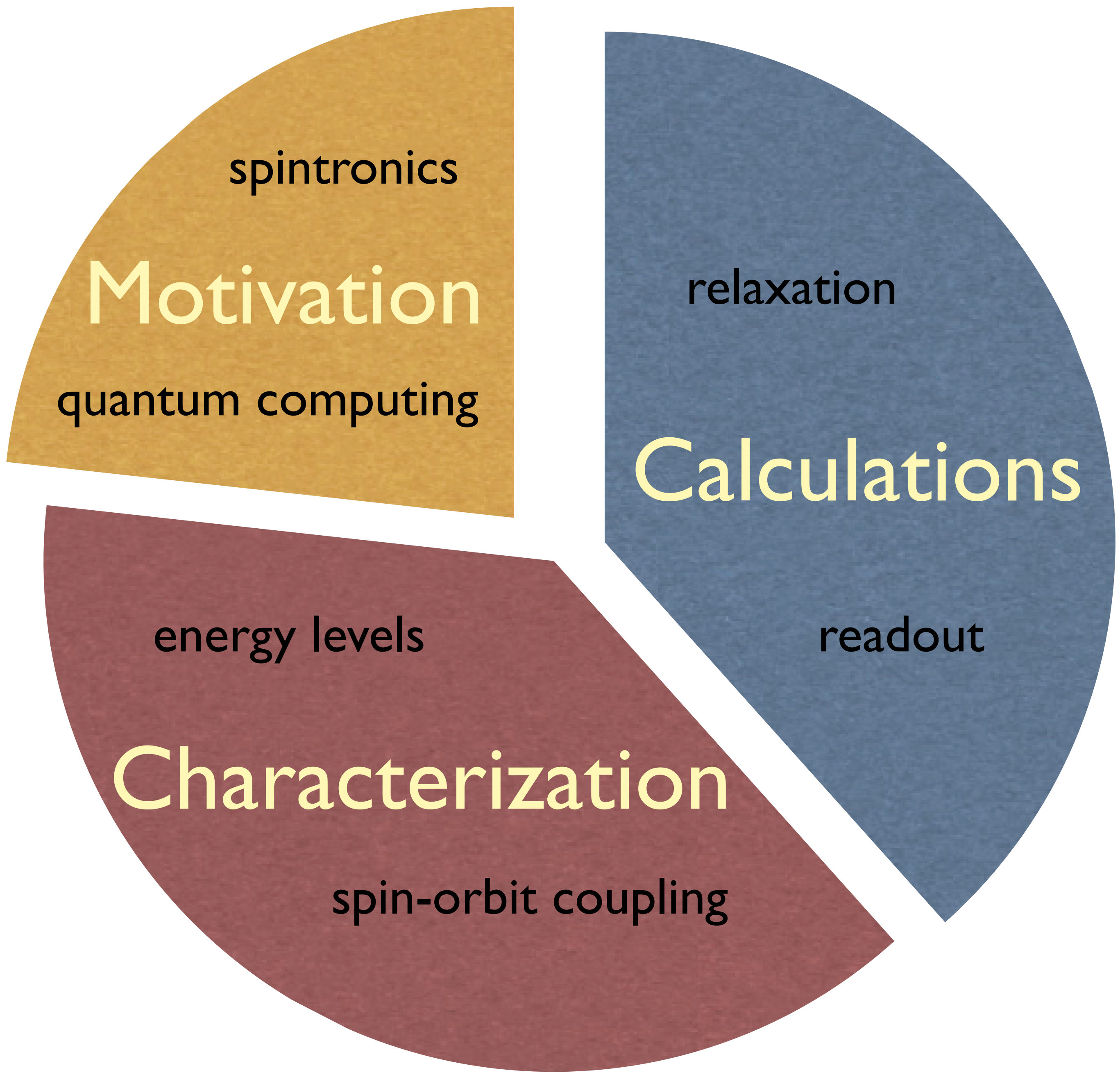
Spring, 2003

qc.physics.wisc.edu

QC Dream Team

Talk Outline

Spin transitions in silicon quantum dots



C. Tahan
qc.physics.wisc.edu

Motivation

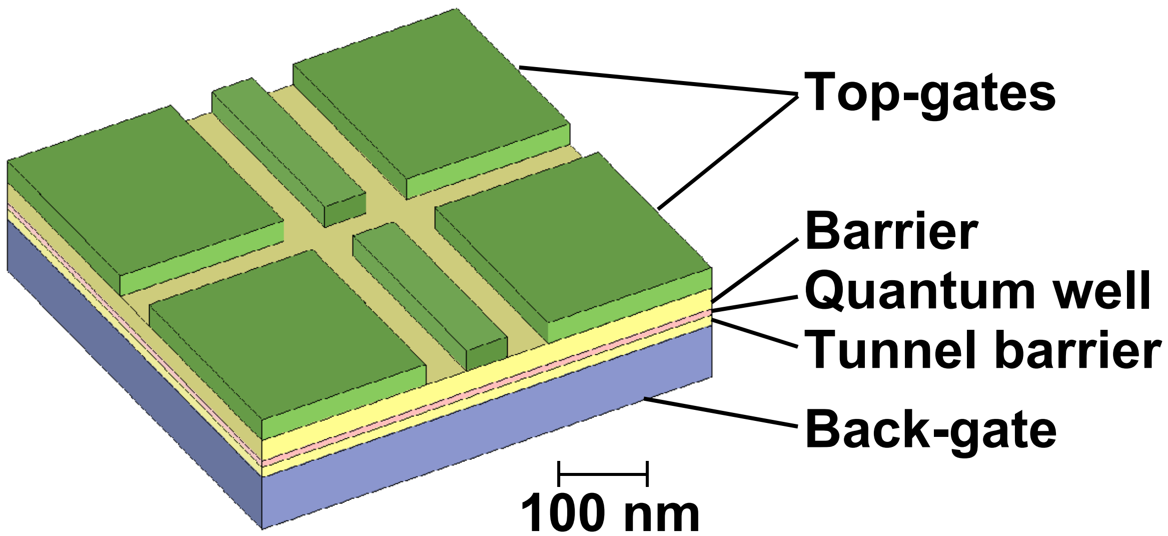
- Quantum

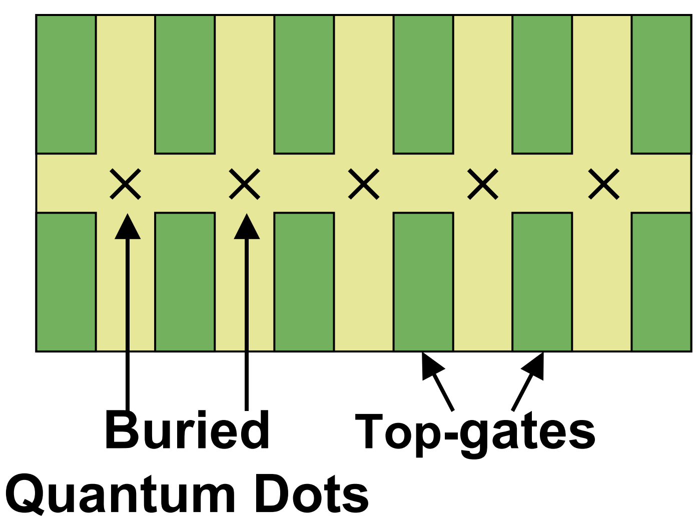
  Coherence
  Information Theory
  Computing
  Physics

- Technology

  Wisconsin QDQC -SiGe
  Spintronics/Spin Transport
  Nanoscale devices

Wisconsin Quantum Dot Quantum Computer



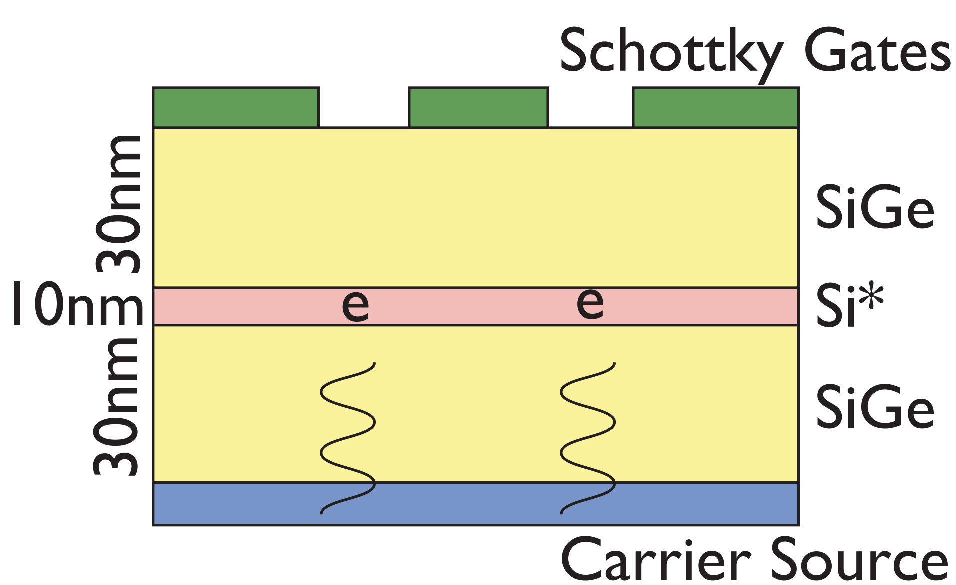


Setup...

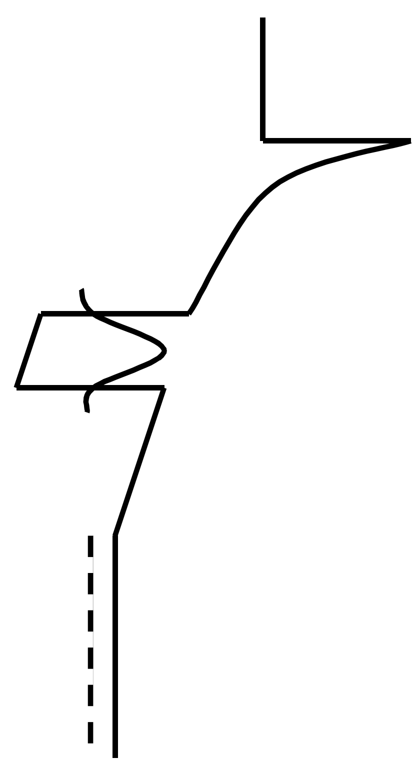
- SiGe/Strained Si/SiGe
  Quantum Well Quantum Dot
- Single electron spin as qubit
- Heisenberg exchange for entangling
- Very low temperatures (100mK ?)

Along the way...

- Master SiGe techonology
- Coherently manipulate spins
- Measure a single spin
- Outrun decoherence
- Test Quantum Mechanics
- ...
- Discover new physics?



Si*=Strained, Isotopically pure



C. Tahan

Spin transitions in
silicon quantum dots

qc.physics.wisc.edu

Quantum Nanodevices

- Pentium VI = Strained Si, Ge
- Spintronics
- Quantum communication

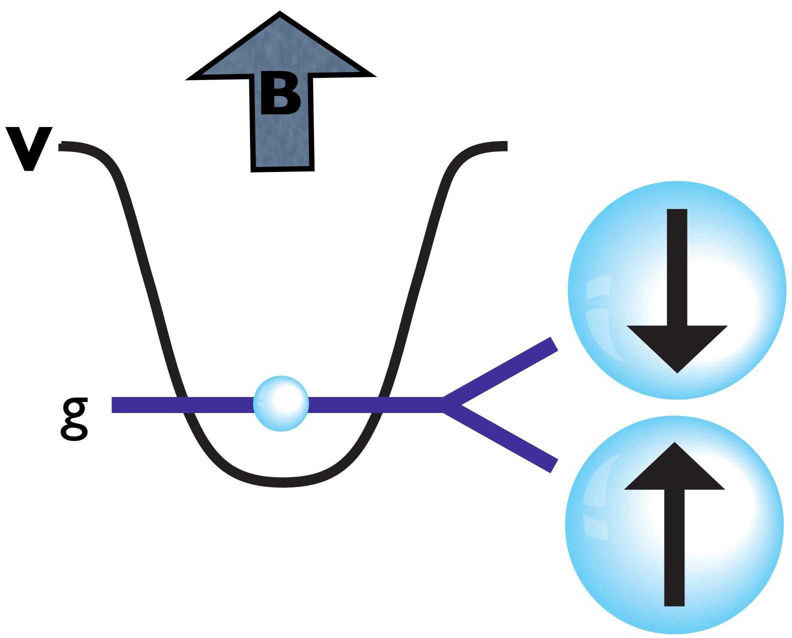
  (spin info to phonon polarization)

Quantum Coherence, A short introduction

state vector formalism $H|\psi\rangle = E|\psi\rangle$ $|\psi\rangle$, pure state

density matrix formalism $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

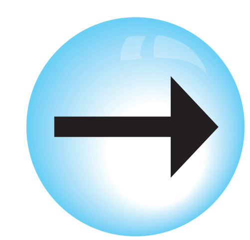
Density matrices describe a quantum system that isn't completely known.



$= |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\rho_{-z} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$= |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\rho_{+z} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

SO,

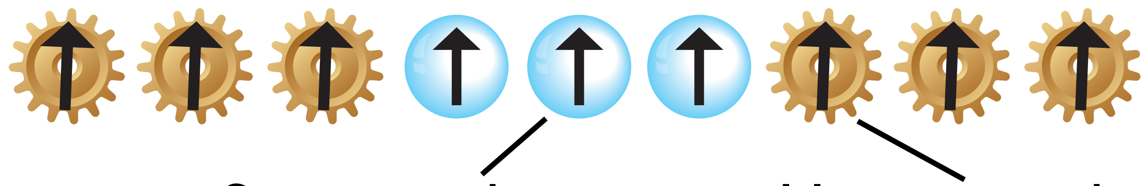


$\rho_x = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

pure vs. mixed states

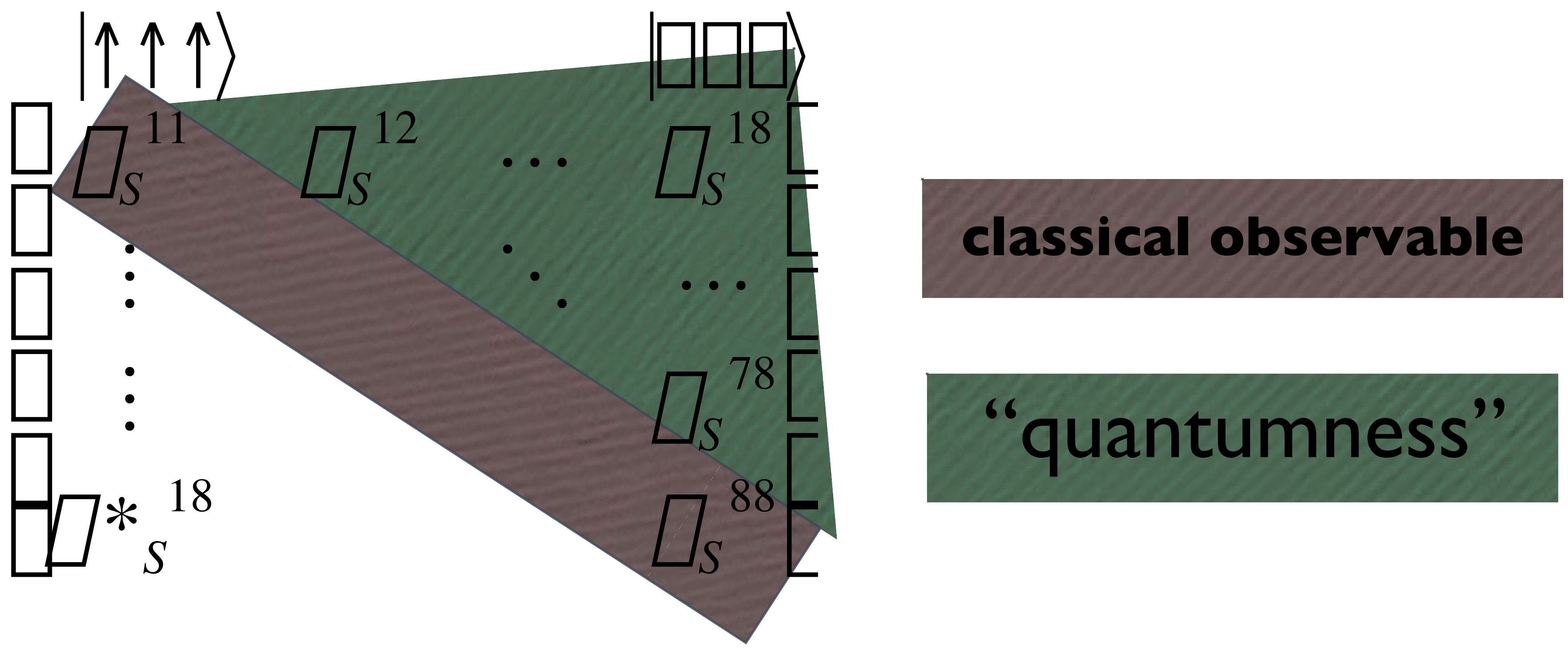
Quantum Coherence, A short introduction, p.2

3 qubit QC uncoupled with the environment (initialized).



$= \rho_S \quad \rho_U$

$\rho_S = \rho_1 \quad \rho_2 \quad \rho_3 =$



The (system + universe) evolve under a Hamiltonian, **H**.

$$\rho_{S+U}(t) = e^{-iHt/\hbar}(\rho_S \quad \rho_U)e^{iHt/\hbar}$$

They become **entangled** and can no longer be expressed as a tensor product.

$$\rho^S = tr_U\left[U\left(\rho^S \quad \rho^U\right)U^\dagger\right]$$

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

Decoherence is the loss of system information to the universe.

Back to a single spin qubit...

Assumptions...

- Markoffian dynamics
- NMR-like
- Phenomenological

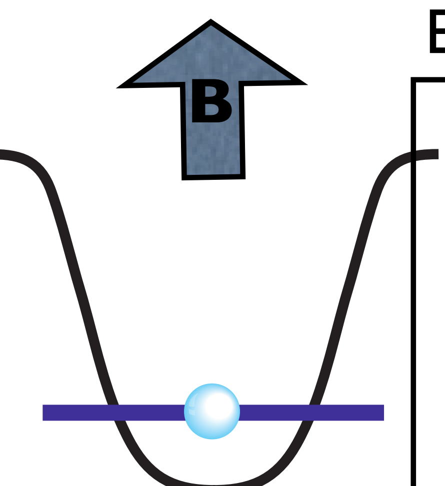
$$\begin{pmatrix} a & b \\ b^* & 1-a \end{pmatrix} \rightarrow \begin{pmatrix} (a-a_0)e^{-t/T_1} + a_0 & be^{-t/2T_2} \\ b^* e^{-t/2T_2} & (a-a_0)e^{-t/T_1} + 1 - a_0 \end{pmatrix}$$

characterization

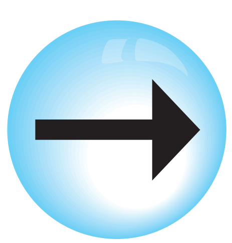
T_1 = classical relaxation time

T_2 = quantum decoherence time

Example: QD e- Spin (T=0K), $T_1 >> T_2$

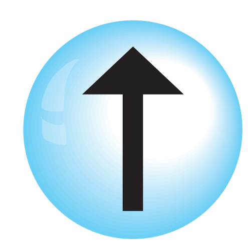


$t = 0$ | $T_1 > t > T_2$ | $t > T_1$



$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$



$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

C. Tahan

qc.physics.wisc.edu

Why T1?

- Path to T2
- Lower limit on decoherence
- T1 ~ T2?
- Wisconsin Readout
- SO coupling

T1 and T2, SoTA Measurements

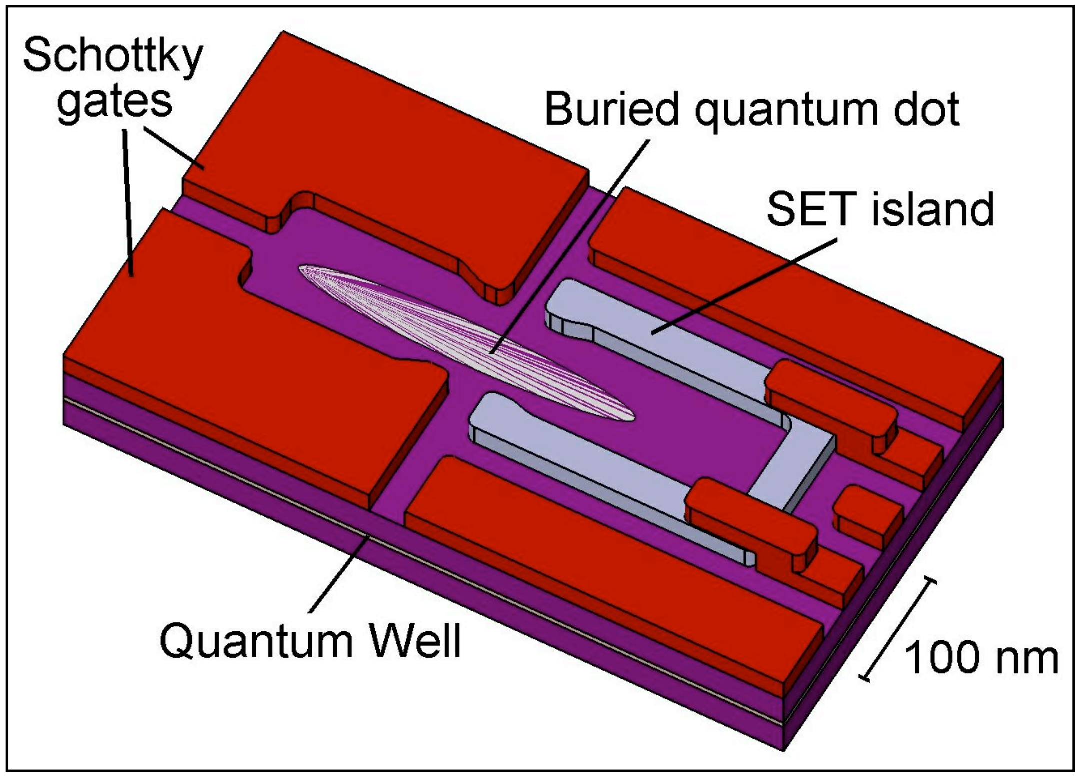
T1: π - T - π/2 - t - π - t - echo
T2: π /2 - t - π - t - echo

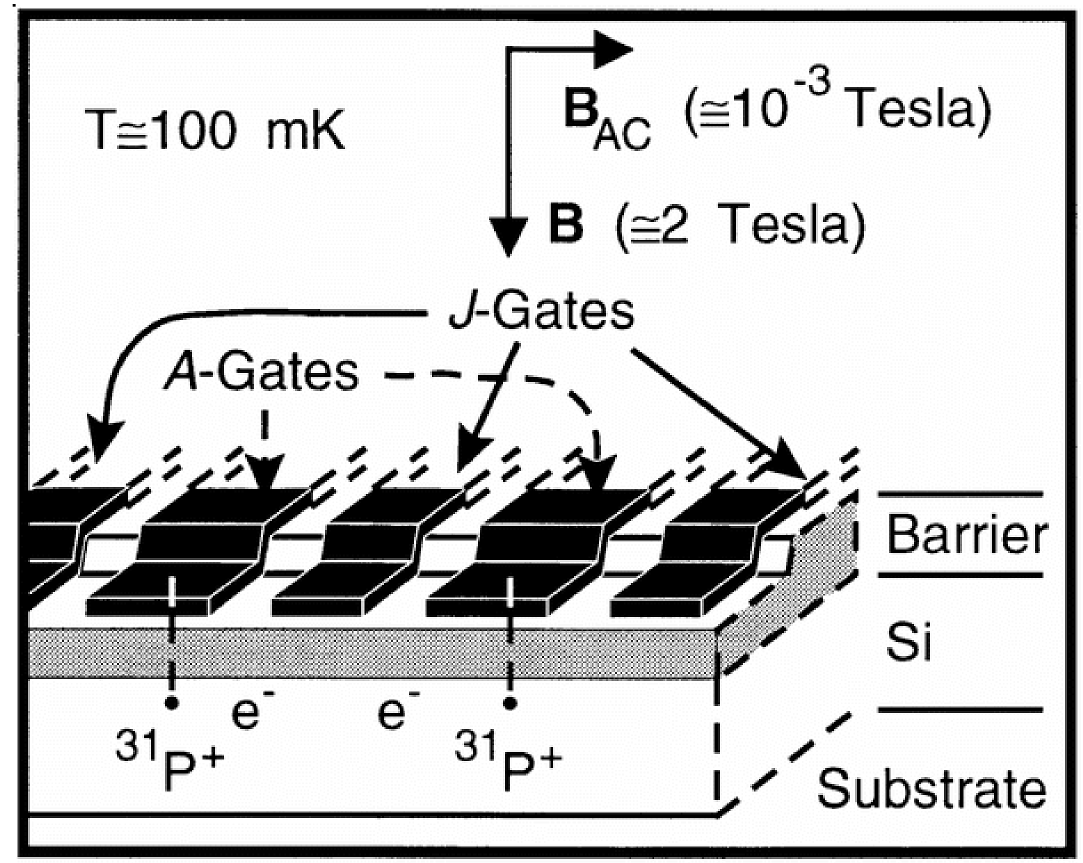
Who	Comments			
PR '61 Feher/ Wilson	Bulk P:Si	n=10^15 P/cm^3 T=1.25K	T1(B=0.3 T) ~ hours T1(B=0.8 T, [110])=800 s T1(B=0.8 T, [100])=1000 s	
J.P.Japan '72 Chiba/Hirai	Bulk P:Si Pulsed ESR T=1.6K	n=10^16 P/cm^3 n=10^17 P/cm^3 n=10^18 P/cm^3	Tm~ 200 micros Tm ~ 5 micros Tm ~ o	spectral diffusion spin-spin
'03 Tyryshkin/ Lyon	T=7-20K Bulk P:Si Isotopically purified	n=10*^15 P/cm^3 n=10*^16 P/cm^3	T1(B~1T) ~ 0.1 s Tm~ 3 ms Tm~ 0.3 ms	dipole-dipole >10^13/cm^3
Si/Si.75Ge.25 2DEGs	Schaffler Wilamowski/Jantsch Tyryshkin/Lyon	n=10*^12 /cm^2 T~5K	T1 ~ T2 ~ micros	

Characterization

- ☑ P:Si Donors vs. Quantum Dots
- ☑ Spin-orbit coupling in heterostructures
- ☐ Phonons and deformation theory in strained silicon

QD vs. P:Si donor

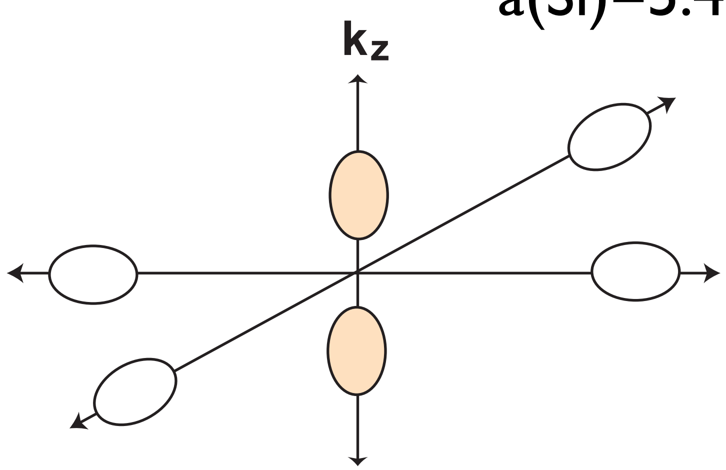


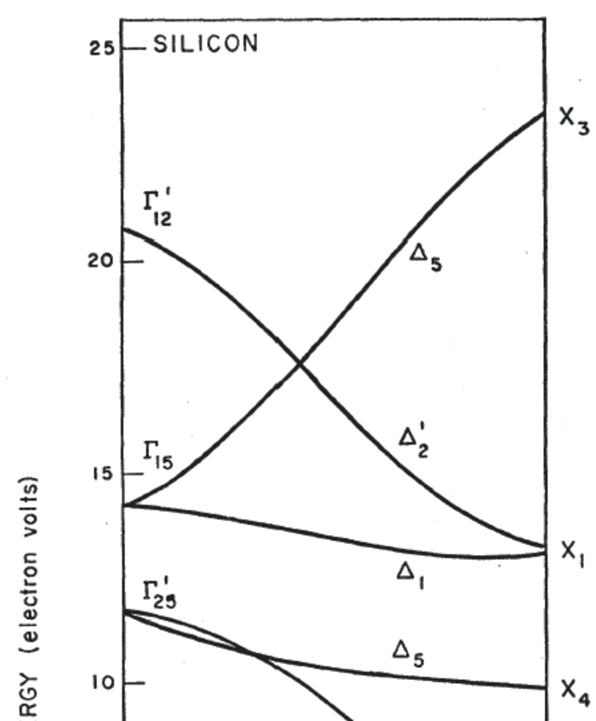


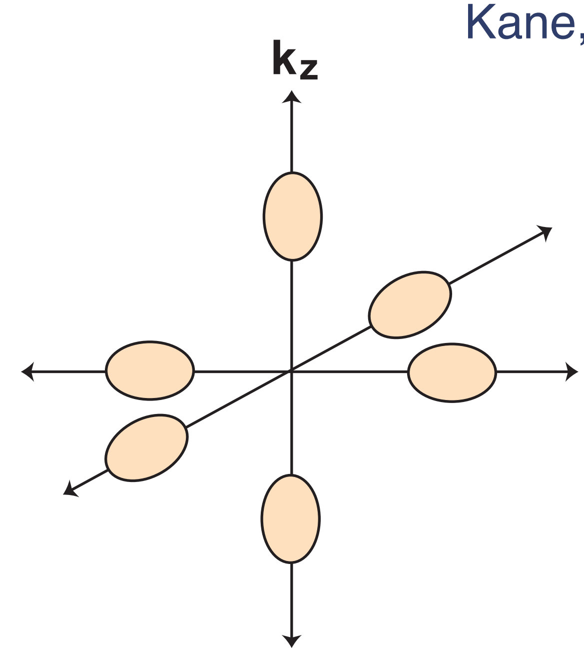
Kane, Nature '98

a(Si.75Ge.25)=5.4825 Å

a(Si)=5.4310 Å







Spin transitions in silicon quantum dots

C. Tahan

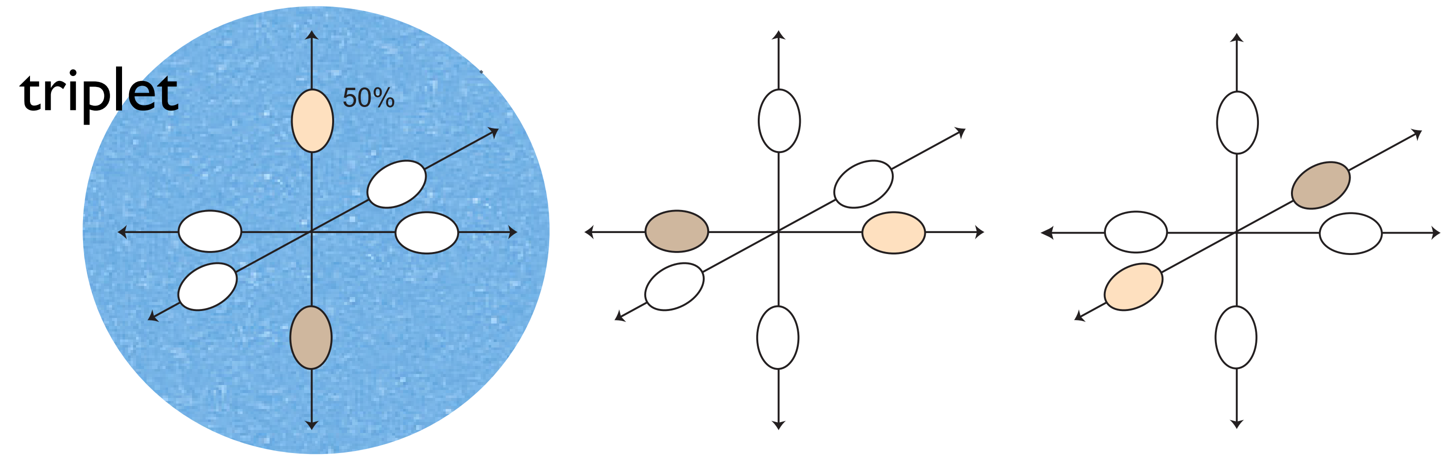
qc.physics.wisc.edu

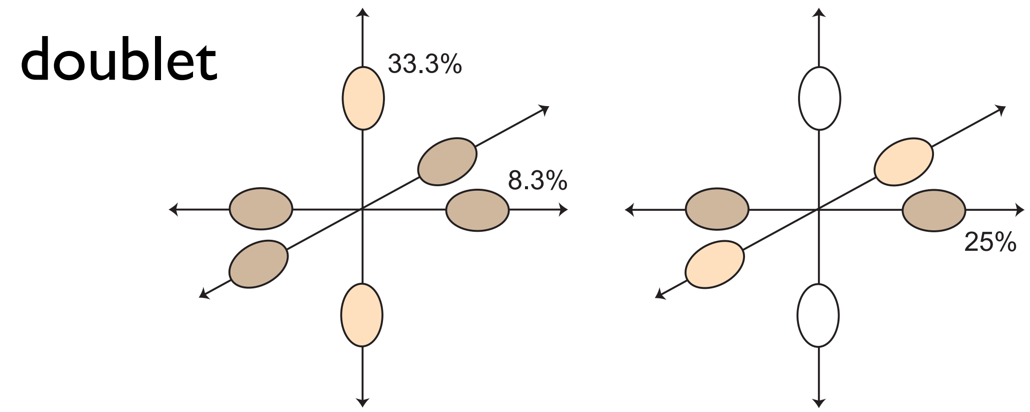
QD vs. P:Si donor

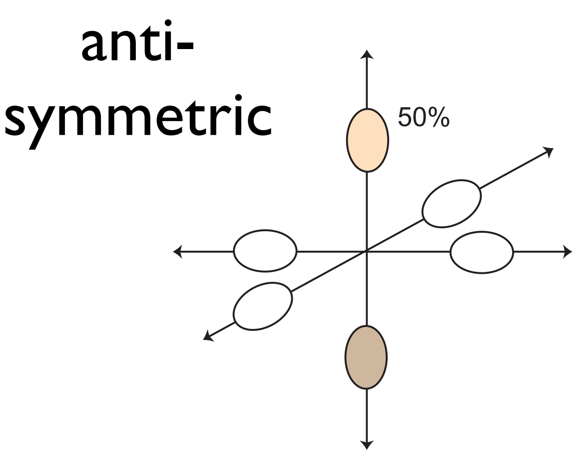
$$\psi = \sum_i C^{(i)} \phi^{(i)}$$

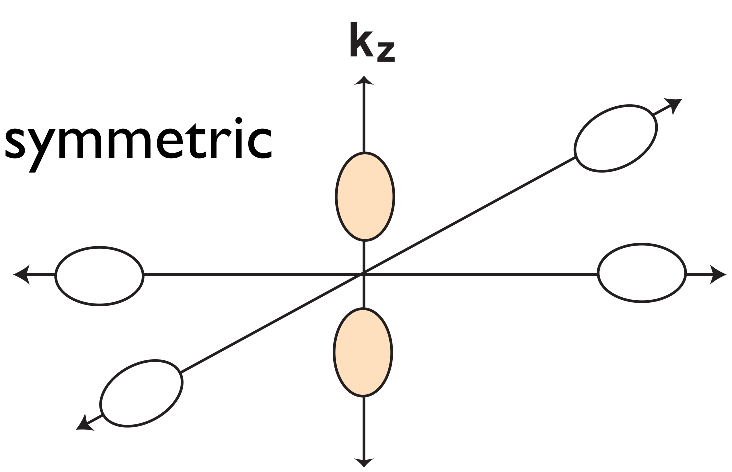
Kohn&Luttinger '57

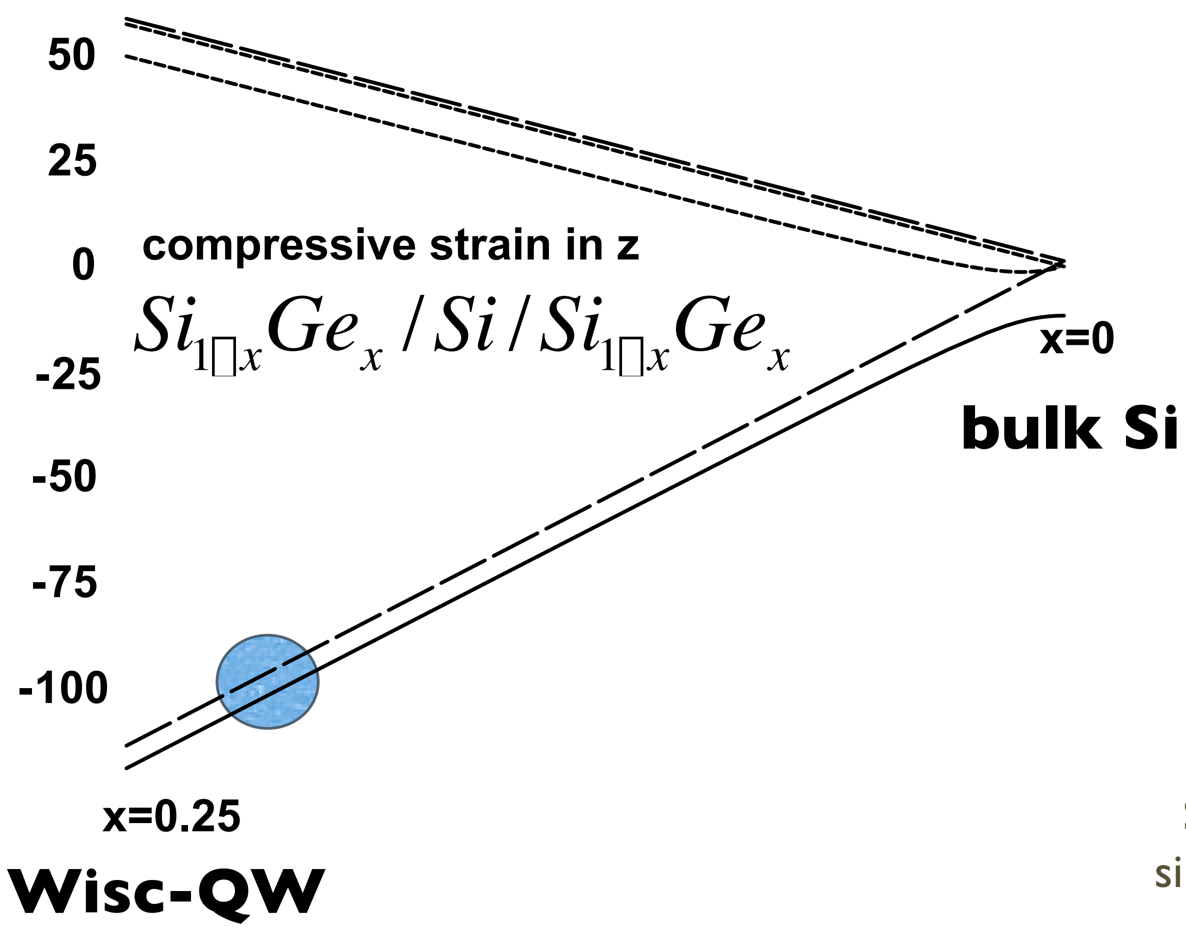
$|C_i|^2$ = % of electron at ith minima

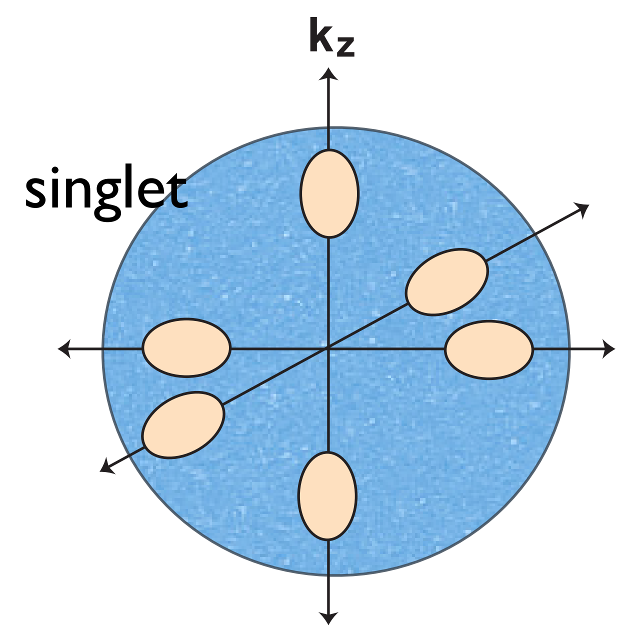










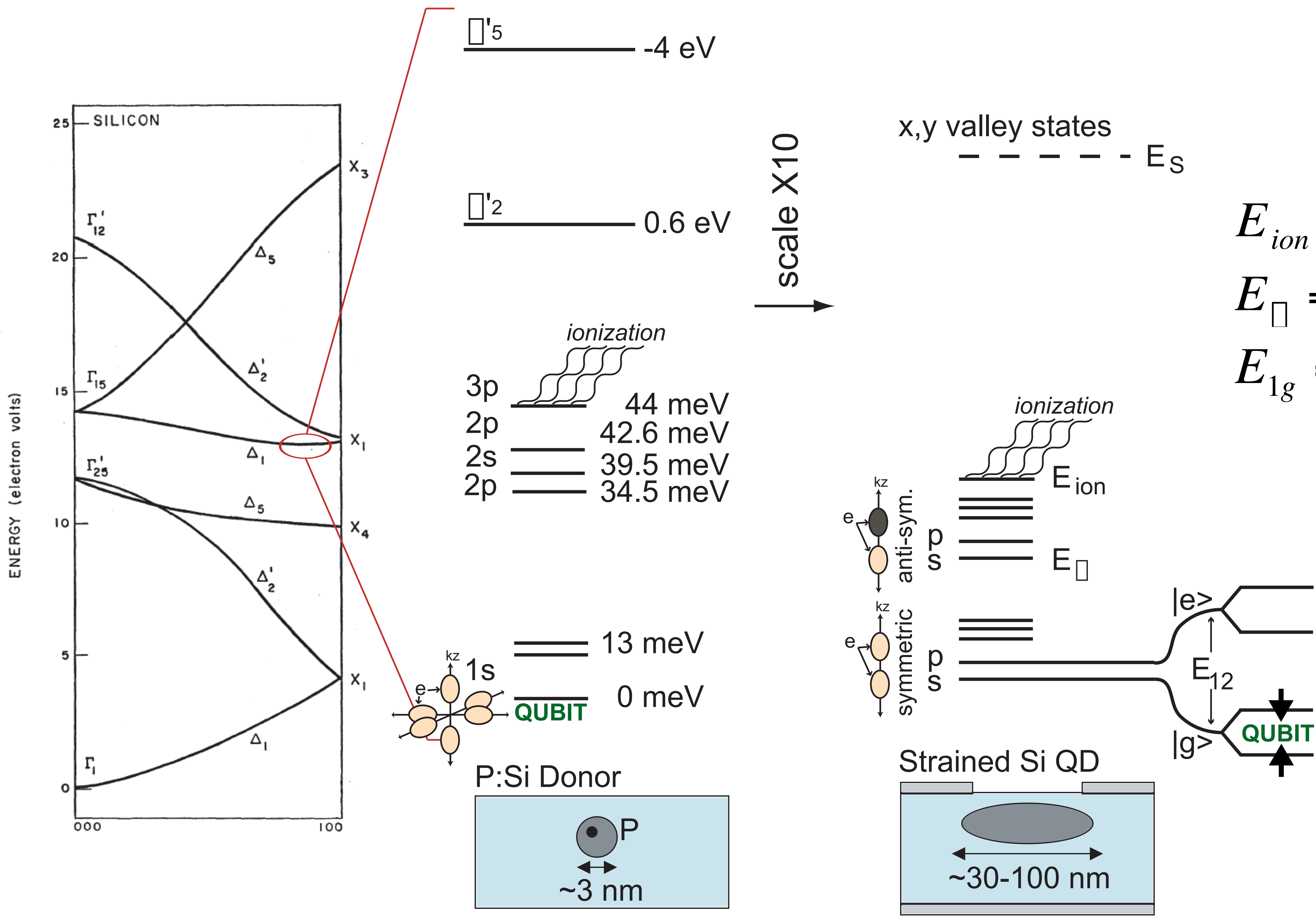


Spin transitions in silicon quantum dots

C. Tahan

qc.physics.wisc.edu

QD vs. P:Si donor, Energy levels



$E_{ion} \sim 4meV(?)$

$E_{□} = 1\text{ meV }(?)$

$E_{1g} = 0.1□0.5\text{ meV}$

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

SO Coupling

Relativistic **free** electron → Dirac equation → Spin-orbit coupling

$$H_{SO} = \frac{\hbar}{4m^2c^2}\vec{\nabla} V \cdot (\vec{\sigma} \times \mathbf{p})$$

enhanced by band effects

bulk crystal + interface or external E-field $\quad V = V(r)_{crystal} + V_{asymmetry}$

A very strong electric field can contribute significantly to spin–orbit coupling.

SO in Si:

$$\Delta g_{\parallel} = -.003$$
$$\Delta g_{\perp} = -.004$$

Rashba:

$$H_R = \alpha(z)\left(p_x\sigma_y - p_y\sigma_x\right)$$

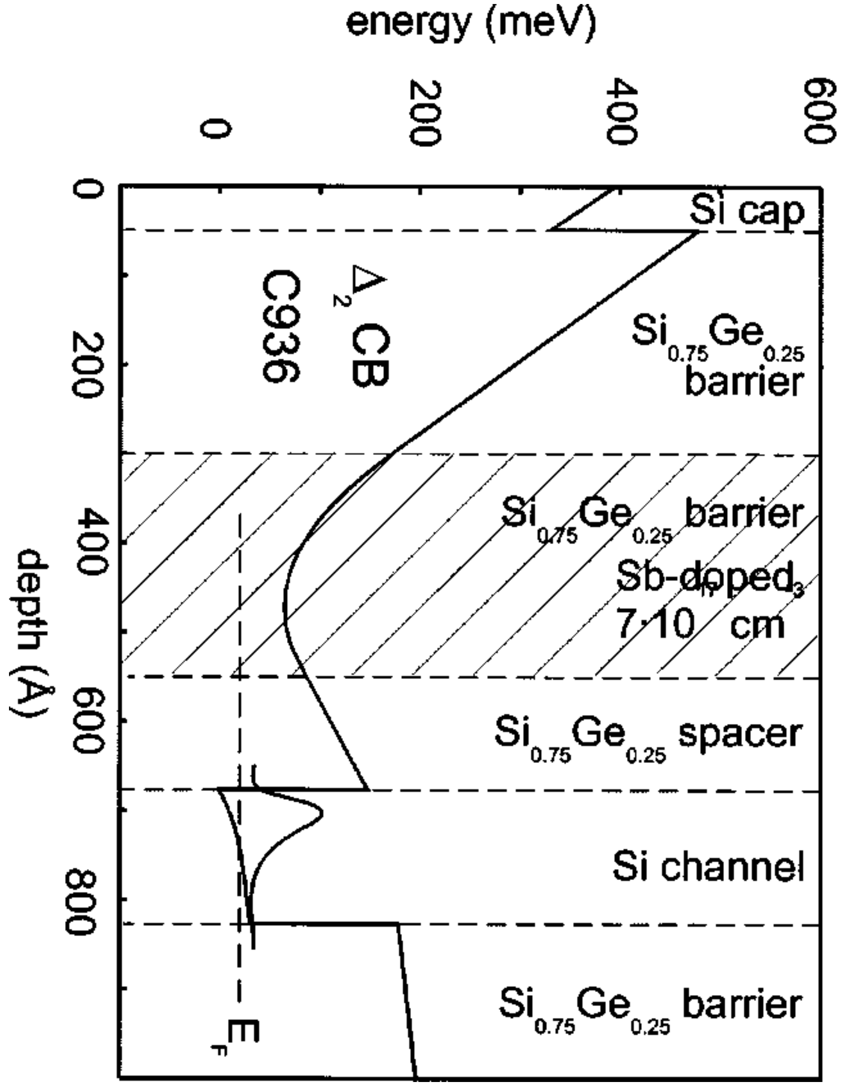
Rashba, Sov.Phys.SS ‘60
Bychkov&Rashba, J.Phys.C ‘84

$$\alpha \propto \mu \frac{dV}{dz}$$

SO Coupling, Heterostructures

Pure Si Quantum Well 2DEG

Wilamowski, Jantsch, Malissa, Rossler, PRB '02



\+ + +

E

\- - -

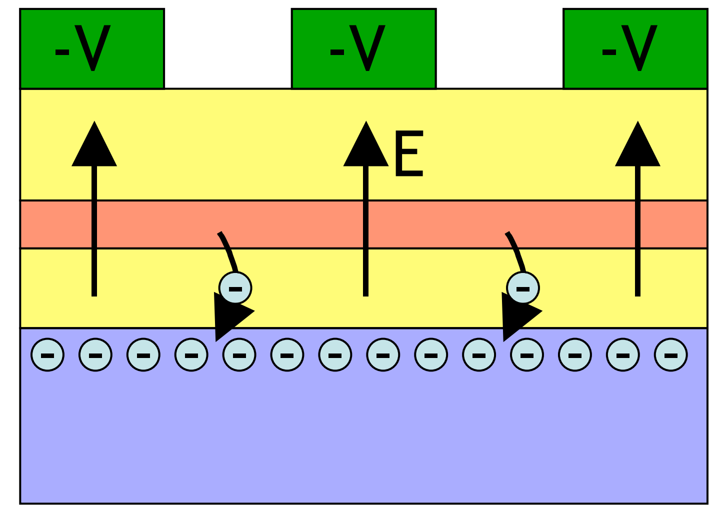
Donor layer + + + + + + +

2DEG e-

$$|\vec{E}| = \frac{e n_{2DEG}}{2\epsilon_0 \epsilon_{Si}} \approx 3 \times 10^6 \text{ V/m}$$

$$n_{2DEG} \approx 4 \times 10^{11} cm^{-2}$$

Wisconsin Proposal I



Top gates

Quantum well

Back gate

$$|\vec{E}| \approx (0.1 - 0.5) \times 10^6 \text{ V/m}$$

Friesen, p.c.

gates - - - -

$\vec{E}$

image charge - - - - - - -

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

SO Coupling, Rashba

SiGe/Si/SiGe 2DEG
with x=0.25

$$\alpha = 8.4\ \text{m/s}$$

High-res. conduction electron spin-resonance
Wilamowski, Jantsch, Malissa, Rossler, PRB ’02
Schaffler sample

$$\alpha \approx \frac{\hbar}{2m^*}\frac{\Delta}{E_g}\frac{2E_g+\Delta}{\left(E_g+\Delta\right)\left(2E_g+2\Delta\right)}e\left\langle E(z)\right\rangle$$

GaAs
de Andrada e Silva *et.al.* PRB ‘97

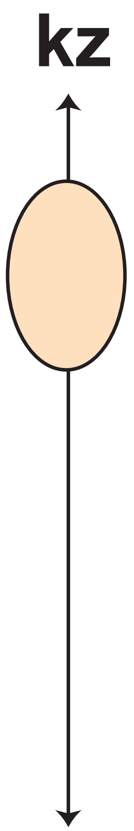
$$=$$

$$\left\langle E(z)\right\rangle \approx 6\times 10^6\ \text{V/m}$$

$$E_g^{direct}(Si_{1-x}Ge_x) = 1.11 - 0.4x\ \text{(eV)}$$
$$E_g^{indirect}(x = .25) = 3.2\ \text{eV}$$
$$\Delta_{SO} = 0.044\ \text{eV}$$
$$m^* = 0.19m$$

C. Tahan

Silicon and Phonon Facts



$$\mathbf{g}^{[001]} = \begin{bmatrix} g_\perp = 1.998 & 0 & 0 \\ 0 & g_\perp = 1.998 & 0 \\ 0 & 0 & g_\parallel = 1.999 \end{bmatrix}$$

$$\mathbf{m}^{[001]} = \begin{bmatrix} m_\perp = 0.19m & 0 & 0 \\ 0 & m_\perp = 0.19m & 0 \\ 0 & 0 & m_\parallel = 0.98m \end{bmatrix}$$

$$H_{electron\,phonon} = \sum_{ij} U_{ij} \Xi_{ij}$$

deformation potential

$$\Xi^{[001]} = \begin{bmatrix} \Xi_d & 0 & 0 \\ 0 & \Xi_d & 0 \\ 0 & 0 & \Xi_d + \Xi_u \end{bmatrix}$$

shear

$\Xi_u \approx 10$ eV

$\Xi_d \approx -11$ eV; 1.1 eV

dilation

Shift in energy of the band edge per unit elastic strain.

strain tensor

$$U_{ij}(\mathbf{q},t)^{phonon} = \frac{i}{2}\sqrt{\frac{\hbar}{2\rho v_t q}}\left[\left(\mathbf{e}(t)_i q_j + \mathbf{e}(t)_j q_i\right) a^\dagger_{\mathbf{q},t} \exp\left(-i\mathbf{q}\cdot\mathbf{r}\right) + c.c.\right]$$

Wavevector **q**, polarization t.

Key points…

- T ~100mK
- No optical phonons
- Acoustic phonons, transverse & longitudinal

$v_l = 9330$ m/s

$v_t = 5420$ m/s

$\rho = 2330$ kg/m^3

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

Calculations

- ☑ Qubit relaxation in QDs
- ☐ Readout:
  - ☑ Zero B-field relaxation
  - ☐ Photon excitation in a QD

T1, Overview

Prior work:

- P:Si donors:
  T<2K: Direct acoustic phonons

  Pines, Slichter, Abrams
  Roth/Hasegawa
  Wilson/Feher

- QDs:
  GaAs, Direct piezophonons via bulk SO mixing

  Khaetskii/Nazarov

New work:

- Rashba SO mixing is dominant mechanism

T1, Valley-repopulation mechanism

1) phonon = time-dependent shear strain

2) singlet and doublet ground states mix

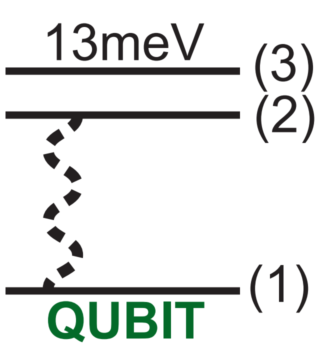
$$\Box = \sum_i C^{(i)} \Box^{(i)}$$

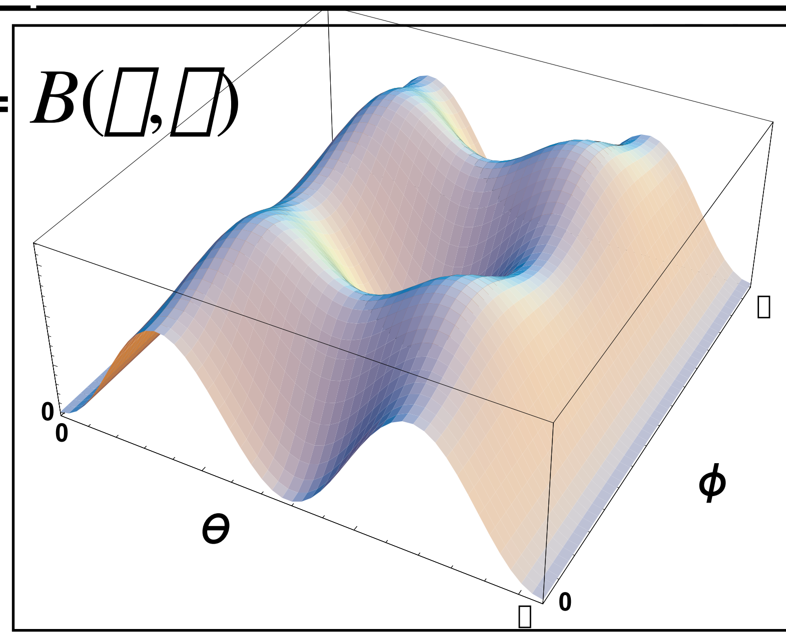
$$M \sim \left\langle \Box_g^{(i)} \middle| \sum_i C_g^{(i)} C_r^{(i)} \Box^{(i)} \middle| \Box_r^{(i)} \right\rangle$$

$$\mathbf{g} = \sum_i \underline{C^{(i)}} \mathbf{g}^{(i)}$$

P:Si donor

$$\vec{B} = B(\Box, \Box)$$

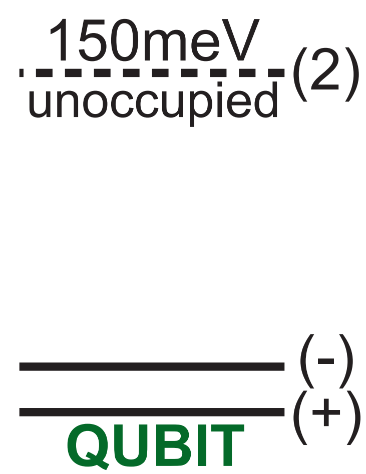




$$\frac{1}{T_1^{repop}} = \frac{1}{\Box \Box_{Si}} \left(\frac{\Box_u}{E_{SD}} \frac{g_{\parallel} \Box g_{\Box}}{3g} \right)^2 \left[2N(g \Box B) + 1\right] \left(\frac{g \Box B}{\hbar} \right)^5 \left(\frac{2}{5 v_t^5} + \frac{4}{15 v_l^5} \right) \left[\sin^2 \Box \cos^2 \Box + \sin^2 \Box \sin^4 \Box \sin^2 \Box \right]$$

QWQD

Tahan *et.al.* PRB '02

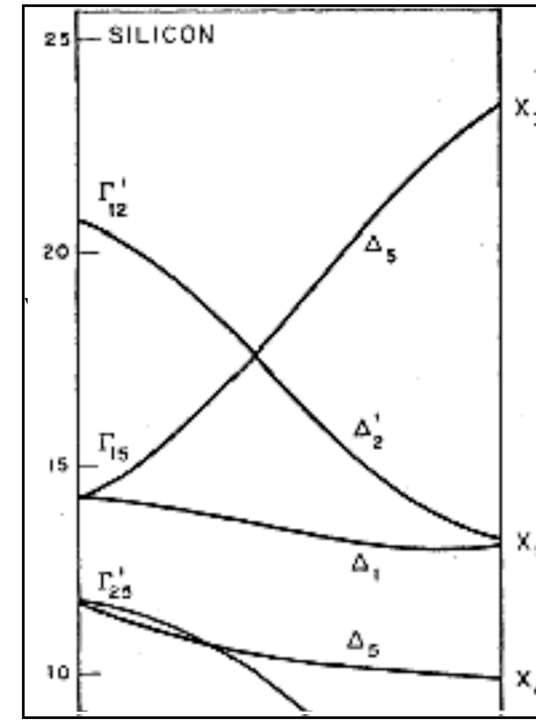


$$C_{\Box} = 1/\sqrt{2}(1,\text{-}1)$$

$$C_{+} = 1/\sqrt{2}(1,\ 1)$$

$$\frac{1}{T_1^{repop}} = 0$$

$\square$ T1, One-valley mechanism



Phonon coupling to nearby conduction bands interband deformation potential

$$\mathbf{g} = \sum_i C^{(i)} \underline{\mathbf{g}^{(i)}}$$

P:Si donor Roth '61

$$H^{bulk} = A\square_B[U_{xy}(\square_x B_y + \square_y B_x) + U_{yz}(\square_y B_z + \square_z B_y) + U_{zx}(\square_z B_x + \square_x B_z)]$$

$$A = \square g_{\square} \frac{E_{15}}{E_{12'}} \langle \square_{2'} | D_{xy} | \square_5 \rangle \square 0.44$$

Wilson&Feher '61

$$\frac{1}{T_1^{one\square v}} = \frac{A^2\hbar}{\square g^2 \square_{Si}} [2N(g\square B)+1] \left(\frac{2}{5v_t^5} + \frac{4}{15v_l^5} \right) \left(\frac{g\square B}{\hbar} \right)^5 \left(\frac{1}{2} \square (\sin^2\square\cos^2\square + \sin^2\square\sin^4\square\sin^2\square) \right)$$

QWQD Glavin&Kim PRB '03

$$H^{QD} = A\square_B U_{xy}(\square_x B_y + \square_y B_x)$$

$$\frac{1}{T_1^{one\square v}} = \frac{A^2\hbar}{32\square g^2 \square_{Si}} [2N(g\square B)+1] \left(\frac{2}{5v_t^5} + \frac{4}{15v_l^5} \right) \left(\frac{g\square B}{\hbar} \right)^5 [\sin^2\square(\cos^2 2\square + \cos^2\square\sin^2 2\square)]$$

T1, one-valley mechanism

QWQD

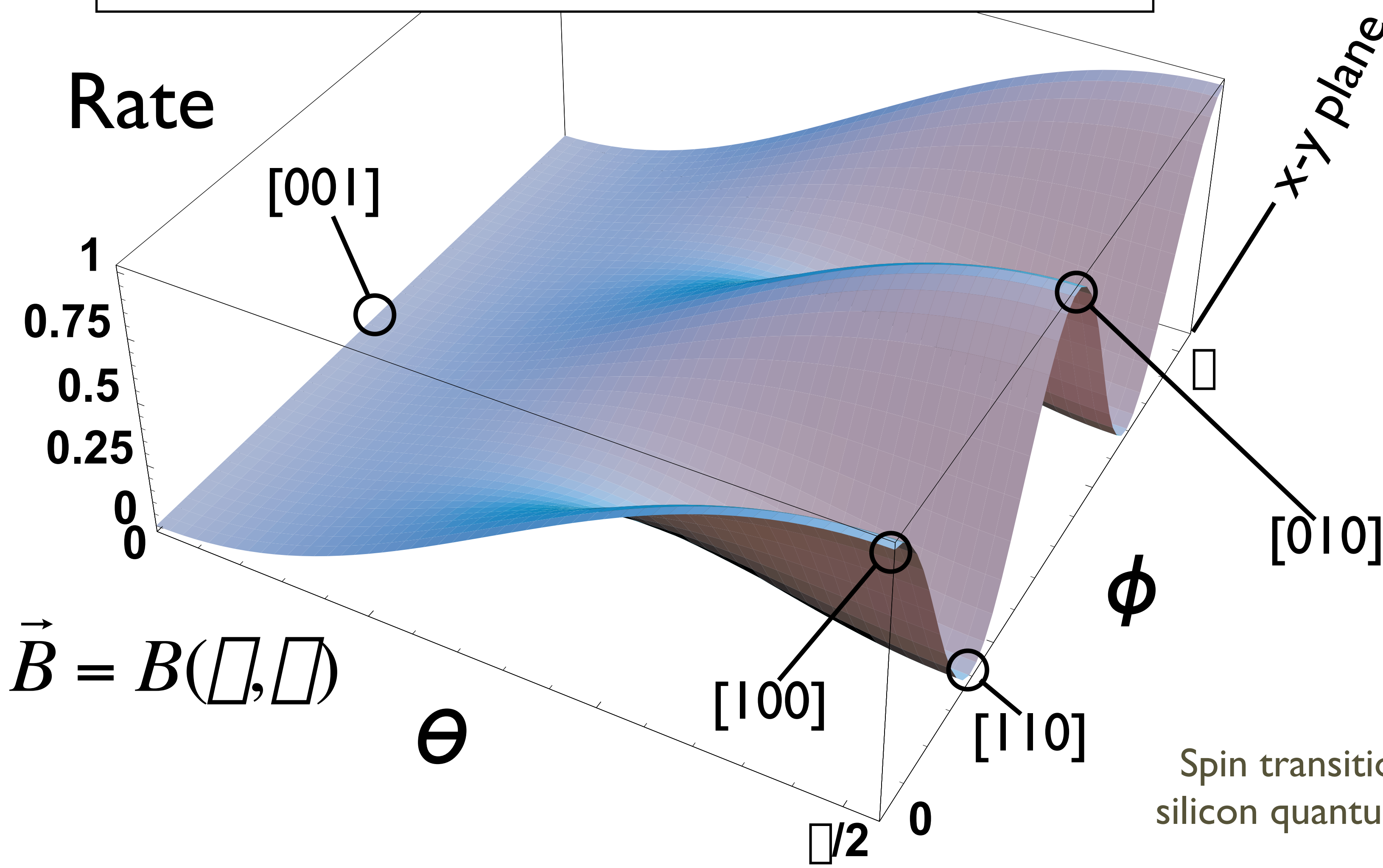
B=0.05 T

$$\frac{1}{T_1^{one-v}}\left(s^{-1}\right) = 1\times10^{-10} f(\theta,\phi)$$

B=2 T

$$\frac{1}{T_1^{one-v}}\left(s^{-1}\right) = 0.01 f(\theta,\phi)$$

$$f(\theta,\phi) = \left[\sin^2\theta\left(\cos^2 2\phi + \cos^2\theta \sin^2 2\phi\right)\right]$$



C. Tahan

Spin transitions in silicon quantum dots

T1, Rashba SO mixing mechanism

QWQD

Spin mixing via higher dot states on the same minima.

Golden Rule transition due to phonon **q**, polarization t.

$$\frac{1}{T_1^R} = \frac{2\pi}{\hbar}\left|\langle 1\uparrow|H_{e-p}|1\downarrow\rangle\right|^2\delta\left(\hbar\omega_{\mathbf{q},t} - g\mu_B B\right)$$

Electron qubit wavefunction, including B-field.

$|n\rangle = |$  $\rangle$ or Fock-Darwin QD states

Include spin-orbit coupling perturbatively.

$$|n\uparrow\rangle_{SO} = |n\uparrow\rangle + \sum_r \frac{|r\rangle\langle r|H_{SO}|n\uparrow\rangle}{E_{nr}} \qquad H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right)$$

$$\frac{1}{T_1^R} = \frac{(m\alpha)^2}{210\rho\hbar}\left(\frac{g\mu_B B}{\hbar}\right)^7\left(\frac{35\Xi_d^{\ 2} + 14\Xi_d\Xi_u + 3\Xi_u^{\ 2}}{v_l^{\ 7}} + \frac{4\Xi_u^{\ 2}}{v_t^{\ 7}}\right)$$
$$\left[\left(\Lambda_{xx} + \Lambda_{yy}\right)\left(3 + \cos 2\theta\right) + \left(\Lambda_{xx} - \Lambda_{yy}\right)\cos 2\phi \sin^2\theta\right]$$

$$\Lambda_{xx} = \sum_r \frac{\langle n|x|r\rangle\langle r|x|n\rangle}{E_{nr}}$$

To first order.

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

T1, Rashba SO mixing mechanism

QWQD

B=0.05 T

$T_R^{[001]} = 1 \times 10^{7}$ s

$T_R^{[100]} = 6 \times 10^{7}$ s

$T_R^{[010]} = 1 \times 10^{7}$ s

$T_R^{[110]} = 2 \times 10^{7}$ s

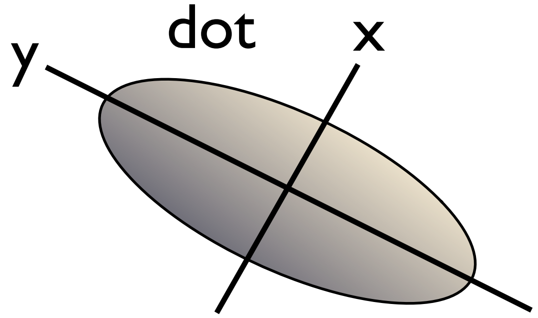
B=2 T

$T_R^{[001]} = 6 \times 10^{-5}$ s

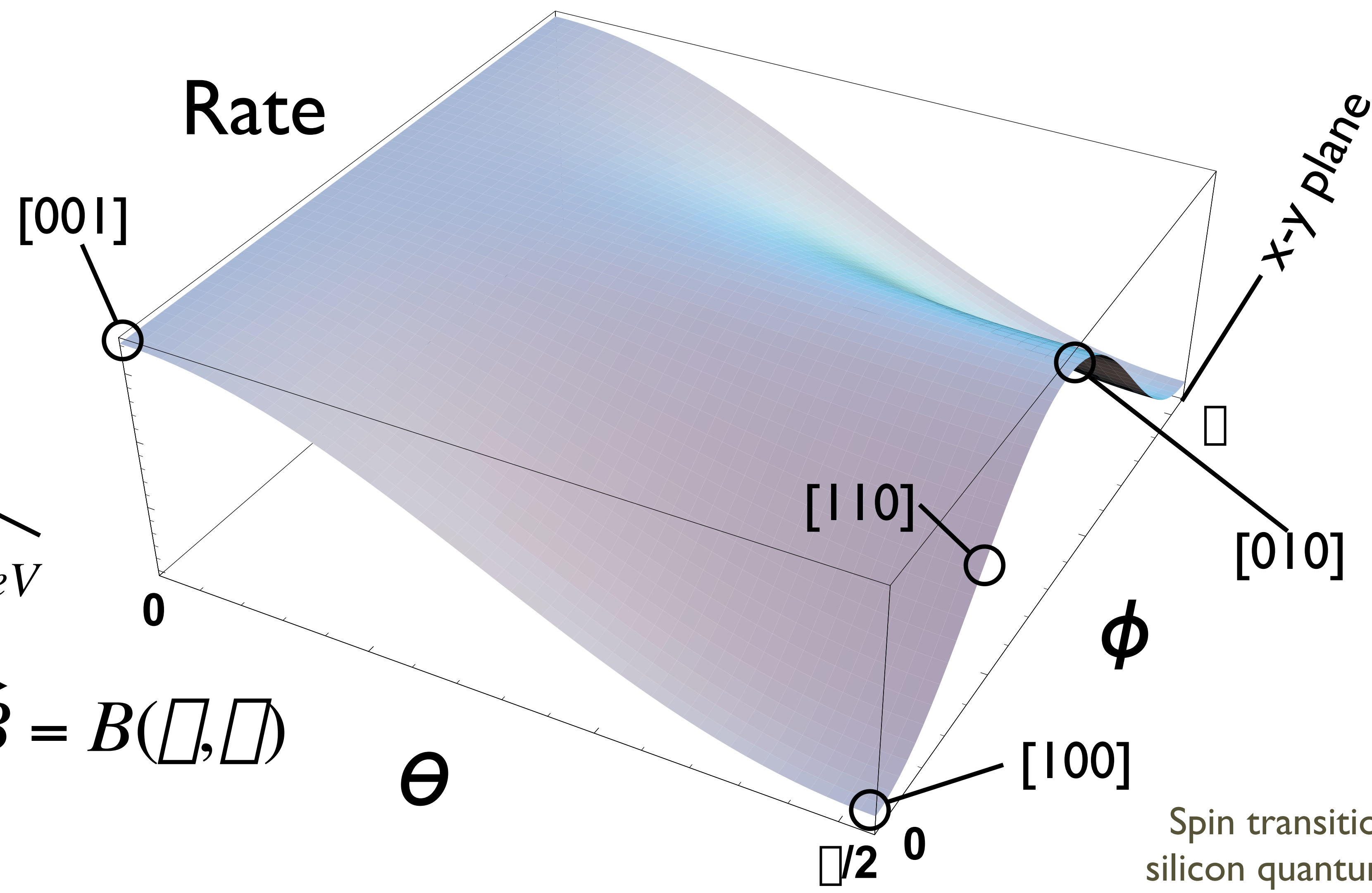
$T_R^{[100]} = 4 \times 10^{-4}$ s

$T_R^{[010]} = 7.5 \times 10^{-5}$ s

$T_R^{[110]} = 1.2 \times 10^{-4}$ s



$(2/3)\hbar\omega_x = \hbar\omega_y = 0.2meV$



$\vec{B} = B(\theta,\phi)$

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

T1, Comparison

Relaxation Rate (1/s)

Field Direction	P:Si donor	One-Valley	Rashba*
[001]	0.006*B^5	0	125*B^7
[110]	0.004*B^5	0	62*B^7
[100]	0.006*B^5	0.0004*B^5	20*B^7
[010]	0.006*B^5	0.0004*B^5	104*B^7

* Assumes parabolic dot, E=0.2meV

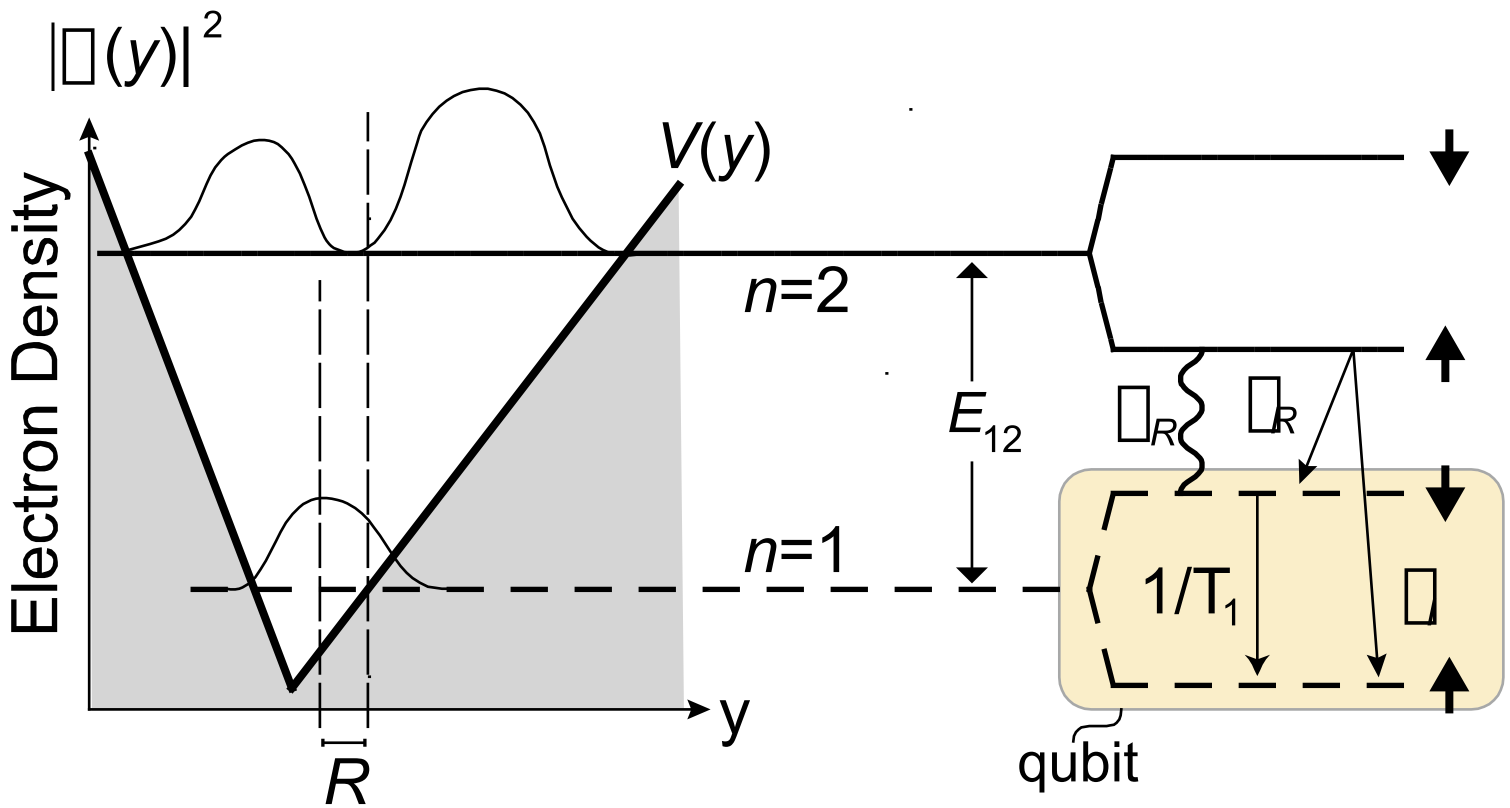
$B_c < 0.01$ T

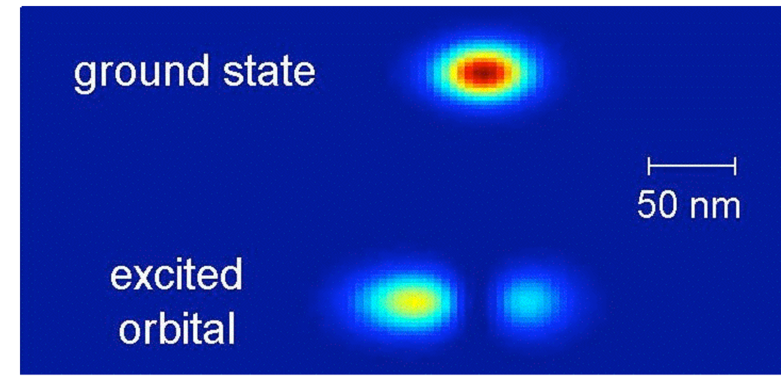
Take Home...

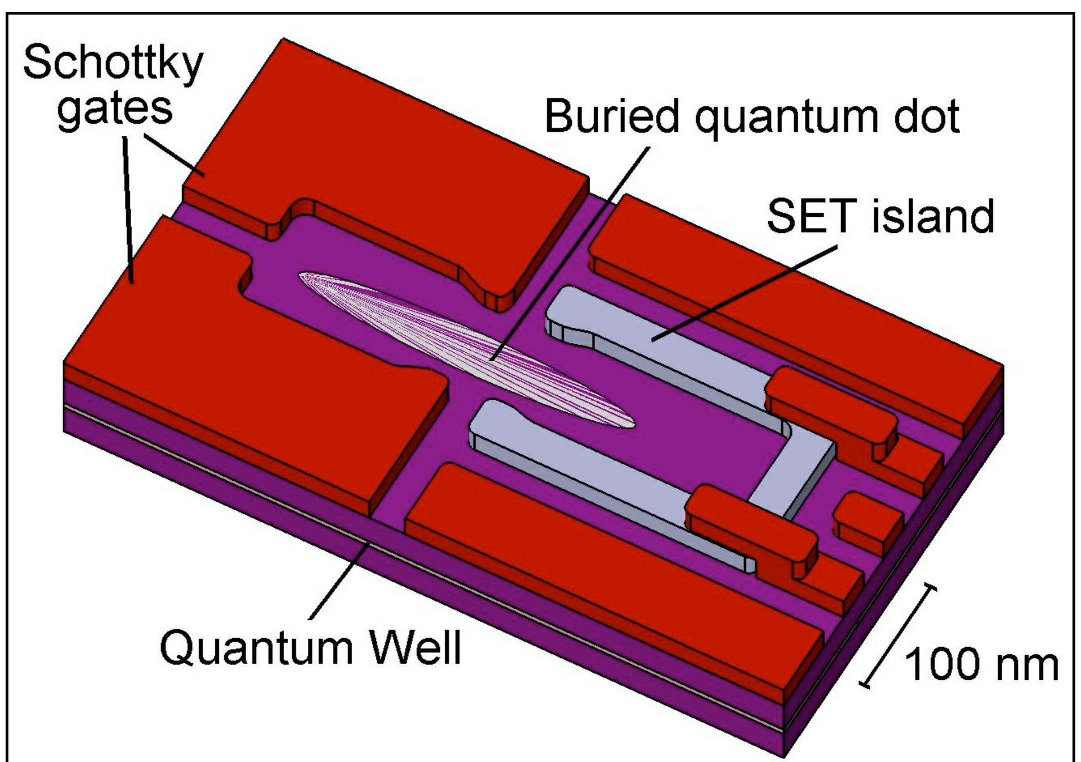
- New mechanism of relaxation due to Rashba SO mixing
- B^7 rate dependence
- Scales as alpha^2
- Very dependent on dot structure
- T1 is still quite long at low B-fields

Wisconsin Readout

Readout, Overview of Wisconsin scheme





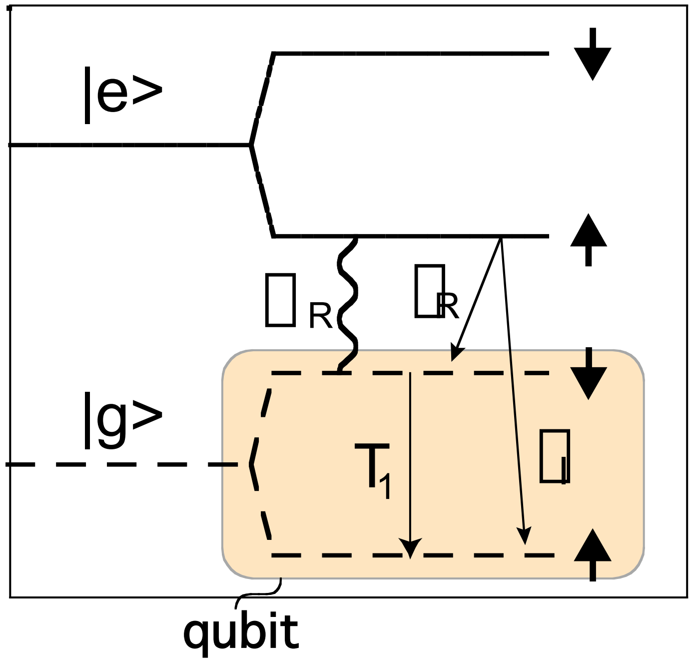


Need...

- Relaxation (Initialization) time
- Rabi frequency (Readout rate)

C. Tahan

Readout, Phonon relaxation (Initialization)



$$\square_I >> \square_R > 1/T_1$$

$$\square_I^{\uparrow\uparrow} = \frac{2\square}{\hbar}\left|\langle g\uparrow|H_{e\square p}|e\uparrow\rangle\right|^2 \square\left(\hbar\square_{\mathbf{q},t} \square E_{eg}\right)$$

$$\square_I^{\uparrow\uparrow} = \frac{E_{eg}^{5}}{\square\hbar^{6}\square}\left(\left|\langle g|x|e\rangle\right|^2 + \left|\langle g|y|e\rangle\right|^2\right)\square\frac{35\square_d^{2} + 14\square_d\square_u + 3\square_u^{2}}{210 v_l^{7}} + \frac{2\square_u^{2}}{105 v_t^{7}}\square$$

Historical footnote: transverse phonons do not contribute in the usual, unstrained bulk Si case for 2p(g)->1s(g) relaxation.

device...

E_{eg} = 0.129 meV

<glyle> = 48 nm

<glxle> = 1.8 nm

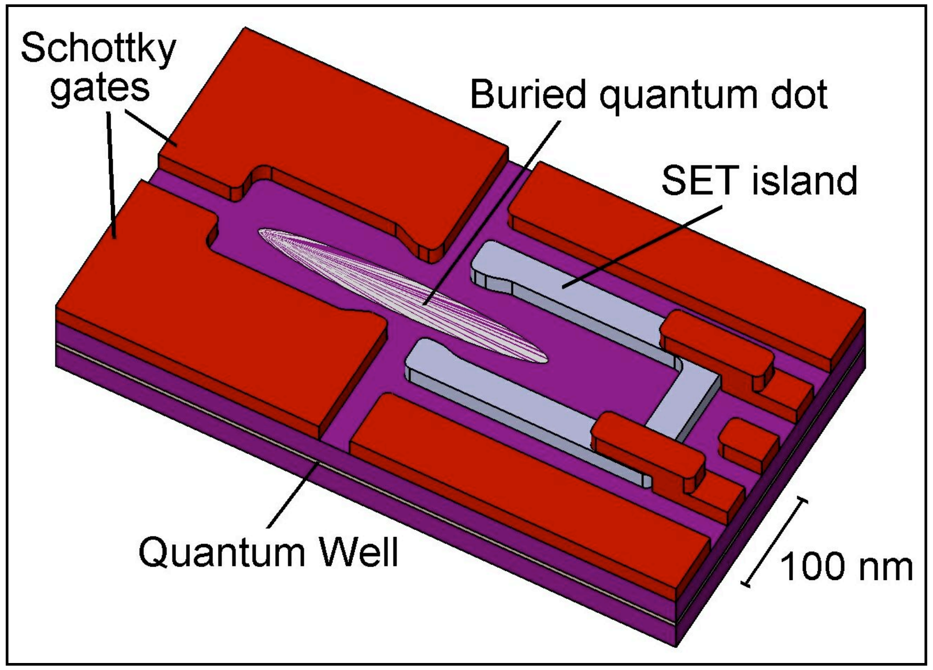
$$\square_I^{\uparrow\uparrow} \square 300\ \text{MHz}\ \ (3\ \text{ns})$$

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

Readout, Photon excitation (Readout)



$$|h\square| = \left|\langle g\square | V_{light} | e\uparrow\rangle\right|$$

$$V_{light} = \square\frac{e}{m}\frac{E_0}{\square_{light}}\square\cdot\mathbf{p} \qquad E_0 = \sqrt{\frac{2Intensity}{c\square_0\sqrt{\square_{Si}}}}$$

$$\square = \left|\square\frac{eE_0}{2\square\left(E_{eg}\square g\square B\right)}\square\cdot\left[\langle g|x\square|e\rangle\square_y^{\uparrow\square}\square\langle g|y\square|e\rangle\square_x^{\uparrow\square}\right]\right|$$

<glx*d/dxle> ~ 0
<gly*d/dyle> = 3.6
<gly*d/dxle> = 0.8
<glx*d/dyle> ~ 0
E_{eg} = 0.129 meV
B=0.05 T (z-dir)

$$\square = \frac{\square eE_0\sqrt{\cos^2\square\cos^2\square + \sin^2\square}}{2\square\left(E_{eg}\square g\square B\right)}\square\square_x\left|\langle g|y\frac{d}{dx}|e\rangle\right| + \square_y\left|\langle g|y\frac{d}{dy}|e\rangle\right|\square$$

$$\square_R = 20{,}000e\left(0.8\square_x + 3.6\square_y\right)\frac{\sqrt{Intensity}}{e\left(0.129\square g\square B\right)}f(\square,\square)$$

C. Tahan

Spin transitions in silicon quantum dots

qc.physics.wisc.edu

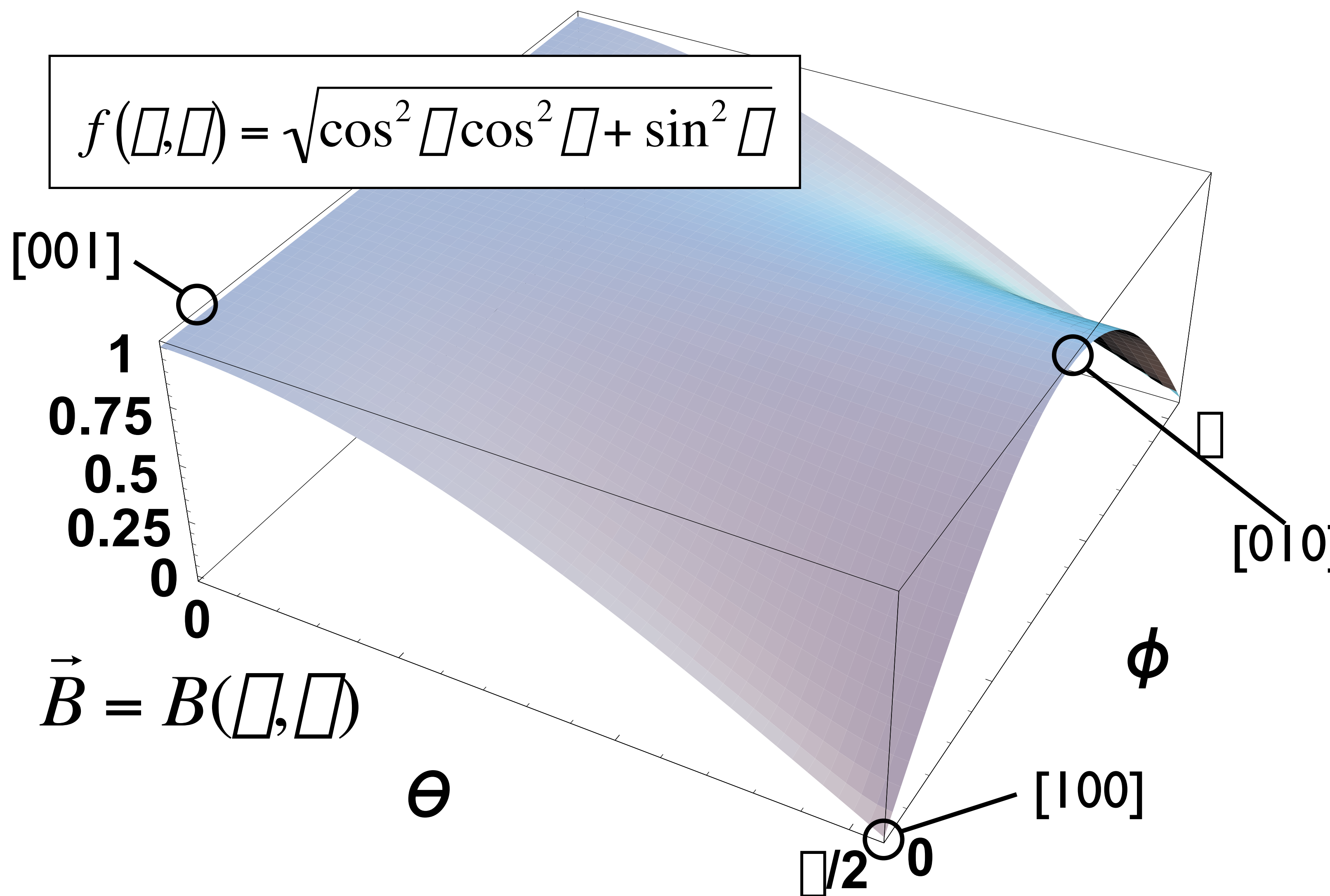
□ Readout, Photon excitation, angular dependence

$$\square_R = 20{,}000e\left(0.8\square_x + 3.6\square_y\right)\frac{\sqrt{Intensity}}{e\left(0.129\,\square\, g\square B\right)}f(\square,\square)$$

$g\square_B(B = 1\text{ T}) = 0.12\text{ meV}$

$Intensity = 0.1\,\square ?\ \ \text{W/m}^2$

$f(\square,\square) = \sqrt{\cos^2\square\cos^2\square + \sin^2\square}$



$\vec{B} = B(\square,\square)$

B=0.05 T

$\square_R^{[001]=[010]}\ \square 0.6\sqrt{Intensity}\ \text{MHz}$

B=1 T

$\square_R^{[010]}\ \square 8\sqrt{Intensity}\ \text{MHz}$

$\square_R^{[100]}\ \square 0\ \text{MHz}$

Spin transitions in
silicon quantum dots

C. Tahan

Summary

- Asymmetric SOC is very important in Wisc. QDQC
- Rashba SO dominates T1 in a Si QWQD at low T
- T1 is still quite long
- Spin control: Very efficient initializer; potential readout scheme
- Followups...